## Contents

Pre	eface		page xv
1	Quanti	zed electromagnetic field and coherent state representations	1
	1.1	Quantization of the electromagnetic field	1
	1.2	State space for the electromagnetic field – Fock space and Fock states	5
	1.3	Quadratures of the field	6
	1.4	Coherent states	7
	1.5	Mixed states of the radiation field	12
	1.6	Diagonal coherent state representation for electromagnetic fields –	
		P-representation	15
	1.7	The Wigner function for the electromagnetic field	18
	1.8	Bosonic systems with finite mass – coherent states and phase-space	
		representations	22
	Exerc	ises	24
	Refere	ences	26
2	Noncla	ssicality of radiation fields	28
	2.1	The Mandel $Q_{\rm M}$ parameter	28
	2.2	Phase-dependent measure of nonclassicality – squeezing parameter $S$	29
	2.3	Single-mode squeezed states – squeezed vacuum	31
	2.4	Squeezed coherent state	37
	2.5	Other measures of nonclassicality	39
	2.6	Mixed nonclassical states – degradation in squeezing	43
	Exerci	ises	45
	Refere	ences	47
3	Two-m	ode squeezed states and quantum entanglement	49
	3.1	The two-mode squeezed states	49
	3.2	Nonclassicality of the two-mode squeezed vacuum	50
	3.3	Quantum phase-space distributions and quadrature distributions	52
	3.4	Cauchy-Schwarz inequalities for nonclassicality in two-mode states	54
	3.5	Conditional measurements on the two-mode squeezed vacuum	55
	3.6	Quantum entanglement in the two-mode squeezed vacuum	56
	3.7	Peres-Horodecki separability criterion for continuous variable systems	56
	3.8	Generation of two-mode nonclassical and entangled states – optical	
		parametric down-conversion	58

viii

Contents

	3.9	Parametric amplification of signals	6
	3.10	Type-II optical parametric down-conversion – production of entangled	
		photons	6
	3.11	Four-photon entanglement using optical parametric down-conversion	6
	3.12	Two-mode mixed nonclassical states	6
	3.13	Entanglement in two-mode mixed Gaussian states	6
	3.14	Application of entanglement to the teleportation of a quantum state	6
	3.15	Nonclassical fields in optical fibers	6
	Exerc		7
	Refer	ences	7
4	Non-G	aussian nonclassical states	7
	4.1	Schrödinger cat state and the cat paradox	7
	4.2	Photon-added and -subtracted states	8
	4.3	Single-photon-added coherent and thermal states	8
	4.4	Squeezing and sub-Poissonian properties of single-photon-added states	8
	4.5	Experimental realization of photon-added nonclassical non-Gaussian	
		states	8
	4.6	Single-photon-subtracted states	8
	4.7	Single-photon-subtracted two-mode states with vortex structure	9
	4.8	Pair-coherent states	9
	Exerc		10
	Refer	ences	10
5	<b>Optica</b>	interferometry with single photons and nonclassical light	10
	5.1	Transformation of quantized light fields at beam splitters	10
	5.2	Beam splitter transformation equivalent to evolution under a	
		Hamiltonian	10
	5.3	Transformation of states by the beam splitter	10
	5.4	Transformation of photon number states by a beam splitter	10
	5.5	Single photons at beam splitters	10
	5.6	Pairs of single photons at beam splitters	10
	5.7	Generalization of the Hong–Ou–Mandel interference to N photons	
		from both ports of the beam splitter	10
	5.8	Transformation of a two-mode squeezed state by a 50-50 beam	
	<b>.</b> 0	splitter	10
	5.9	Generation of two-mode entangled states by the interference of	
	£ 10	coherent fields and single photons	11
	5.10	Beam splitter as an attenuator	11
	5.11 5.12	Transformation of quantized light fields by phase shifters The Mach–Zehnder interferometer	11
	J. 12	THE WACH-Lennuer Interferometer	11

117

117

5.13 Wheeler's delayed choice gedanken experiment

5.14 Interaction-free measurements

Contents

	5.15	Two-photon Mach–Zehnder interferometer	119		
	5.16	Multiphoton interference and engineering of quantum states	12		
	5.17	Mach–Zehnder interferometer with two-mode squeezed vacuum as			
		input	123		
	5.18	Balanced homodyne interferometers for measuring the squeezing			
		of light	12:		
	5.19	Manipulation of quantum states by homodyning and feed-forward	120		
	5.20		128		
	5.21		129		
	5.22	Heisenberg limited sensitivity of interferometers based on parametric			
		amplifiers or four-wave mixers	13		
	5.23	The quantum statistics of fields at the output ports	133		
	Exerc		134		
	Refer	ences	130		
	Polariz	ration and orbital angular momentum of quantum fields	138		
	6.1	Characterization of the polarization properties of quantized fields	13		
	6.2	Polarization of quantized fields – Stokes operators	139		
	6.3	Action of polarizing devices on quantized fields	14		
	6.4	Description of unpolarized light beyond Stokes parameters	14:		
	6.5	Stokes operator tomography	14		
	6.6	Orbital angular momentum of fields – HG and LG modes	14		
	6.7	Orbital Stokes operators and the Poincaré sphere	149		
	6.8	Mixed states of orbital angular momentum	15		
	6.9	Entangled states of the orbital angular momentum	15		
	6.10	Transformation of entanglement between polarization and orbital			
		angular momentum q-plates	15		
	Exerc	•	15		
	Refer	ences	15		
,	Absor	otion, emission, and scattering of radiation	15		
	7.1	The interaction of radiation and matter in the electric dipole			
		approximation	15		
	7.2	Rates for the absorption and emission of radiation	15		
	7.3	Single-mode limit – Einstein's B coefficient and the absorption			
		coefficient $\alpha(\omega)$	16		
	7.4	Scattering of radiation	16		
	7.5	Quantum interferences in scattering	16		
	7.6	Radiative decay of states – Weisskopf–Wigner theory	17		
	7.7	Control of spontaneous emission through the design of the			
		electromagnetic vacuum	17		
	Exerc		17		
		References			

Contents

interferometers

8	Partia	l coherence in multimode quantum fields	17		
	8.1	Correlation functions for electromagnetic fields	17		
	8.2	Young's interferometer and spatial coherence of the field	18		
	8.3	Photon-photon correlations - intensity interferometry	18		
	8.4	Higher-order correlation functions of the field	18		
	8.5	Interferometry in the spectral domain	18		
	8.6	Squeezing spectrum and spectral homodyne measurement	19		
	8.7	Coherence effects in two-photon absorption	193		
	8.8	Two-photon imaging – ghost imaging using $G^{(2)}$	19		
	Exerc	rises	19		
	Refer	rences	19		
9	Open	quantum systems	20		
	9.1	Master equation description of open systems	200		
	9.2	Dissipative dynamics of harmonic oscillators	204		
	9.3	Dissipative dynamics of a two-level system	200		
	9.4	Dissipative dynamics of a multilevel system	208		
	9.5	Time correlation functions for multilevel systems	210		
	9.6	Quantum Langevin equations	212		
	9.7	Exactly soluble models for the dissipative dynamics of the oscillator	213		
	9.8	Exact dissipative dynamics of a two-level system under dephasing	21:		
	Exerc		218		
	Refer	ences	219		
10	Ampli	fication and attenuation of quantum fields	220		
	10.1	Quantum theory of optical amplification	220		
	10.2	Loss of nonclassicality in the amplification process	223		
	10.3	Amplification of single-photon states	229		
	10.4	Amplification of entangled fields	230		
	10.5	Realising a phase-insensitive amplifier from a phase-sensitive amplifier	232		
	10.6	Degradation of nonclassicality and entanglement due to the absorption	202		
		of quantum fields	233		
	10.7	Loss of coherence on interaction with the environment	235		
	Exerc		238		
	Refer		240		
11	Quantum coherence, interference, and squeezing in two-level systems 2				
-	11.1	Two-level approximation: atomic dynamics in a monochromatic field	242 242		
	11.2	Application of atomic coherence – Ramsey interferometry	247		
	11.3	Atomic coherent states	249		
	11.4	Minimum uncertainty states for two-level systems – spin squeezing	252		
	11.5	Atomoc squeezed states by nonlinear unitary transformations	254		
	11.6	Atomic squeezed states produced by supersensitivity of Ramsey	232		

256

Contents

		i	
	11.7	Phase-space representation for a collection of two-level systems	258
	11.8	Phase-space description of EPR correlations of spin systems	262
	Exerci	ses	264
	Refere	ences	265
12	•	quantum electrodynamics	267
		Exact solution of the Jaynes-Cummings model: dressed states	268
		Collapse and revival phenomena in JCM	271
	12.3	Dispersive limit of the JCM	273
	12.4	Dissipative processes in cavity QED – the master equation	275
	12.5	Spectroscopy of the ladder of dressed states	277
	12.6	• •	284
	12.7	• • •	
		shift of the vacuum	288
	12.8	Atomic cat states using multi-atom dispersive JCM	290
	12.9	11	293
		Engineering anti-Jaynes-Cummings interaction	296
		QED in coupled cavity arrays – single-photon switch	298
	Exerc		300
	Refere	ences	301
12	Ahcorn	tion, emission, and scattering from two-level atoms	304
13	13.1		304
	13.1		304
	13.2	two-level system	309
	13.3		314
	13.4		317
	Exerc		325
	Refere		327
	Keien	Chicos	321
14	Quanti	um interference and entanglement in radiating systems	328
	14.1	Young's interference with microscopic slits – atoms as slits	328
	14.2	Spatial bunching and antibunching of photons	330
	14.3	Interference in radiation from two incoherently excited atoms	333
	14.4	Atom-photon entanglement	337
	14.5	Atom-atom entanglement via detection of spontaneously emitted	
		photons	338
	14.6	Multi-atom entanglement	341
	14.7	-	343
	14.8	Multi-path quantum interference as the source of Dicke superradiance	345
	14.9	Entanglement of photons produced in an atomic cascade	348
	Exerc		351
	Refer		352

xii Contents

15	Near f	ield radiative effects	35
	15.1	Near field radiative effects – coupling between dipoles	35
	15.2	Radiative coupling between dipoles and dynamics	35
	15.3	Vacuum-induced deterministic entanglement	36
	15.4	Two-photon resonance induced by near field radiative effects	36
	15.5	The dipole blockade	36
	Exerc	<del>-</del>	36
	Refer	rences	36
16	Decoh	erence and disentanglement in two-level systems	37
	16.1	Decoherence due to the interaction of a two-level system with	
		the environment	37
	16.2	Disentanglement in two-level systems	37
	16.3	Decoherence-free subspace	37
	16.4	Protection of decoherence due to dephasing via dynamical decoupling	37
	16.5	Control of the spectral density of environment for protection against	
		decoherence	37
	16.6	Modulation produced protection against disentanglement in cavity QED	38
	Exerc	ises	38
	Refer	ences	38
17	Cahan	nut control of the entirel non-control	
17		ent control of the optical properties	38
	17.1	A simple model for coherent control	38
		Dark states and coherent population trapping	39
		EIT in single-atom fluorescence	39
		Control of two-photon absorption	40
		Vacuum-induced coherence and interference	40
	Exerc Refer		40
	Keier	ences	41
18		sion management and ultraslow light	41
	18.1	Group velocity and propagation in a dispersive medium	41
	18.2	Electromagnetically induced waveguides	41
	18.3	Storage and retrieval of optical pulses	41
	18.4	Adiabatons and storage and retrieval of pulses	42
	18.5	Non-EIT mechanisms for ultraslow light	42
	Exerc		42
	Refer	ences	43
19	Single	photons and nonclassical light in integrated structures	43
	19.1	Quantum optics in a coupled array of waveguides	43
	19.2	The Hong-Ou-Mandel interference in a system of two coupled	
		waveguides	12

Cont
------

1				
		19.3	Single-photon transport and coherent Bloch oscillations in a coupled	
			array	436
	•	19.4	The Anderson localization of quantum fields in coupled waveguide	
			arrays	442
		19.5	Discrete quantum walks via waveguide couplers on a chip	447
		Exerc	ises	452
		Refer	ences	453
	20	Quant	um optical effects in nano-mechanical systems	455
		20.1	The radiation pressure on the nano-mechanical mirror	455
		20.2	Basic quantum Langevin equations for the coupled system of cavity	
			and NMO	457
		20.3	Steady-state solution of quantum Langevin equations in the mean field	
			limit and bistability	459
		20.4	Quantum fluctuations in optomechanical systems	461
		20.5	Sideband cooling of the nano-mechanical mirror	463
		20.6	Normal-mode splitting	466
		20.7	Squeezing of a nano-mechanical oscillator	471
		20.8	Electromagnetically induced transparency (EIT) in the mechanical	457.5
		20.0	effects of light	475
		20.9	Quantized states of the nano-mechanical mirror coupled to the cavity	481
		Exerc		485
		Refer	ences	487
	Inc	lex		489