Contents

1 ■ Special Relativity and Flat Spacetime 1

1.1 Prelude 1
1.2 Space and Time, Separately and Together 3
1.3 Lorentz Transformations 12
1.4 Vectors 15
1.5 Dual Vectors (One-Forms) 18
1.6 Tensors 21
1.7 Manipulating Tensors 25
1.8 Maxwell's Equations 29
1.9 Energy and Momentum 30
1.10 Classical Field Theory 37
1.11 Exercises 45

2 ■ Manifolds 48

2.1 Gravity as Geometry 48
2.2 What Is a Manifold? 54
2.3 Vectors Again 63
2.4 Tensors Again 68
2.5 The Metric 71
2.6 An Expanding Universe 76
2.7 Causality 78
2.8 Tensor Densities 82
2.9 Differential Forms 84
2.10 Integration 88
2.11 Exercises 90

3 ■ Curvature 93

3.1 Overview 93
3.2 Covariant Derivatives 94
3.3 Parallel Transport and Geodesics 102

3.4 Properties of Geodesics 108
3.5 The Expanding Universe Revisited 113
3.6 The Riemann Curvature Tensor 121
3.7 Properties of the Riemann Tensor 126
3.8 Symmetries and Killing Vectors 133
3.9 Maximally Symmetric Spaces 139
3.10 Geodesic Deviation 144
3.11 Exercises 146

4 ■ Gravitation 151
4.1 Physics in Curved Spacetime 151
4.2 Einstein's Equation 155
4.3 Lagrangian Formulation 159
4.4 Properties of Einstein's Equation 165
4.5 The Cosmological Constant 171
4.6 Energy Conditions 174
4.7 The Equivalence Principle Revisited 177
4.8 Alternative Theories 181
4.9 Exercises 190

5 ■ The Schwarzschild Solution 193
5.1 The Schwarzschild Metric 193
5.2 Birkhoff's Theorem 197
5.3 Singularities 204
5.4 Geodesics of Schwarzschild 205
5.5 Experimental Tests 212
5.6 Schwarzschild Black Holes 218
5.7 The Maximally Extended Schwarzschild Solution 222
5.8 Stars and Black Holes 229
5.9 Exercises 236

6 ■ More General Black Holes 238
6.1 The Black Hole Zoo 238
6.2 Event Horizons 239
6.3 Killing Horizons 244
6.4 Mass, Charge, and Spin 248
6.5 Charged (Reissner–Nordström) Black Holes 254
6.6 Rotating (Kerr) Black Holes 261
6.7 The Penrose Process and Black-Hole Thermodynamics 267
6.8 Exercises 272

7 ■ Perturbation Theory and Gravitational Radiation 274
7.1 Linearized Gravity and Gauge Transformations 274
7.2 Degrees of Freedom 279
7.3 Newtonian Fields and Photon Trajectories 286
7.4 Gravitational Wave Solutions 293
7.5 Production of Gravitational Waves 300
7.6 Energy Loss Due to Gravitational Radiation 307
7.7 Detection of Gravitational Waves 315
7.8 Exercises 320

8 ■ Cosmology 323
8.1 Maximally Symmetric Universes 323
8.2 Robertson–Walker Metrics 329
8.3 The Friedmann Equation 333
8.4 Evolution of the Scale Factor 338
8.5 Redshifts and Distances 344
8.6 Gravitational Lensing 349
8.7 Our Universe 355
8.8 Inflation 365
8.9 Exercises 374

9 ■ Quantum Field Theory in Curved Spacetime 376
9.1 Introduction 376
9.2 Quantum Mechanics 378
9.3 Quantum Field Theory in Flat Spacetime 385
9.4 Quantum Field Theory in Curved Spacetime 394
9.5 The Unruh Effect 402
9.6 The Hawking Effect and Black Hole Evaporation 412

APPENDIXES 423

A ■ Maps between Manifolds 423

B ■ Diffeomorphisms and Lie Derivatives 429

C ■ Submanifolds 439

D ■ Hypersurfaces 443

E ■ Stokes's Theorem 453

F ■ Geodesic Congruences 459

G ■ Conformal Transformations 467

H ■ Conformal Diagrams 471

I ■ The Parallel Propagator 479

J ■ Noncoordinate Bases 483

Bibliography 495

Index 501