
Contents

1	Probability	1
1.1	Sample Spaces and Events	1
1.1.1	The Sample Space of an Experiment	1
1.1.2	Events	3
1.1.3	Some Relations from Set Theory	4
1.1.4	Exercises: Section 1.1 (1–12)	5
1.2	Axioms, Interpretations, and Properties of Probability	7
1.2.1	Interpreting Probability	9
1.2.2	More Probability Properties	11
1.2.3	Determining Probabilities Systematically	13
1.2.4	Equally Likely Outcomes	14
1.2.5	Exercises: Section 1.2 (13–30)	14
1.3	Counting Methods	18
1.3.1	The Fundamental Counting Principle	18
1.3.2	Tree Diagrams	19
1.3.3	Permutations	20
1.3.4	Combinations	22
1.3.5	Exercises: Section 1.3 (31–49)	25
1.4	Conditional Probability	29
1.4.1	The Definition of Conditional Probability	30
1.4.2	The Multiplication Rule for $P(A \cap B)$	32
1.4.3	The Law of Total Probability and Bayes' Theorem	34
1.4.4	Exercises: Section 1.4 (50–78)	37
1.5	Independence	43
1.5.1	$P(A \cap B)$ When Events Are Independent	44
1.5.2	Independence of More than Two Events	45
1.5.3	Exercises: Section 1.5 (79–100)	47
1.6	Simulation of Random Events	51
1.6.1	The Backbone of Simulation: Random Number Generators	51
1.6.2	Precision of Simulation	55
1.6.3	Exercises: Section 1.6 (101–120)	56
1.7	Supplementary Exercises (121–150)	60
2	Discrete Random Variables and Probability Distributions	67
2.1	Random Variables	67
2.1.1	Two Types of Random Variables	69
2.1.2	Exercises: Section 2.1 (1–10)	70

2.2	Probability Distributions for Discrete Random Variables	71
2.2.1	A Parameter of a Probability Distribution	74
2.2.2	The Cumulative Distribution Function	75
2.2.3	Another View of Probability Mass Functions	78
2.2.4	Exercises: Section 2.2 (11–28)	79
2.3	Expected Value and Standard Deviation	83
2.3.1	The Expected Value of X	83
2.3.2	The Expected Value of a Function	86
2.3.3	The Variance and Standard Deviation of X	88
2.3.4	Properties of Variance	90
2.3.5	Exercises: Section 2.3 (29–48)	91
2.4	The Binomial Distribution	95
2.4.1	The Binomial Random Variable and Distribution	97
2.4.2	Computing Binomial Probabilities	99
2.4.3	The Mean and Variance of a Binomial Random Variable	101
2.4.4	Binomial Calculations with Software	102
2.4.5	Exercises: Section 2.4 (49–74)	102
2.5	The Poisson Distribution	107
2.5.1	The Poisson Distribution as a Limit	107
2.5.2	The Mean and Variance of a Poisson Random Variable	110
2.5.3	The Poisson Process	110
2.5.4	Poisson Calculations with Software	111
2.5.5	Exercises: Section 2.5 (75–89)	111
2.6	Other Discrete Distributions	114
2.6.1	The Hypergeometric Distribution	114
2.6.2	The Negative Binomial and Geometric Distributions	117
2.6.3	Alternative Definition of the Negative Binomial Distribution	120
2.6.4	Exercises: Section 2.6 (90–106)	120
2.7	Moments and Moment Generating Functions	123
2.7.1	The Moment Generating Function	125
2.7.2	Obtaining Moments from the MGF	127
2.7.3	MGFs of Common Distributions	128
2.7.4	Exercises: Section 2.7 (107–128)	129
2.8	Simulation of Discrete Random Variables	131
2.8.1	Simulations Implemented in R and Matlab	134
2.8.2	Simulation Mean, Standard Deviation, and Precision	135
2.8.3	Exercises: Section 2.8 (129–141)	138
2.9	Supplementary Exercises (142–170)	140
3	Continuous Random Variables and Probability Distributions	147
3.1	Probability Density Functions and Cumulative Distribution Functions	147
3.1.1	Probability Distributions for Continuous Variables	148
3.1.2	The Cumulative Distribution Function	152
3.1.3	Using $F(x)$ to Compute Probabilities	154
3.1.4	Obtaining $f(x)$ from $F(x)$	155
3.1.5	Percentiles of a Continuous Distribution	156
3.1.6	Exercises: Section 3.1 (1–18)	158
3.2	Expected Values and Moment Generating Functions	162
3.2.1	Expected Values	162
3.2.2	Moment Generating Functions	166
3.2.3	Exercises: Section 3.2(19–38)	168

3.3	The Normal (Gaussian) Distribution	172
3.3.1	The Standard Normal Distribution	173
3.3.2	Non-standardized Normal Distributions	175
3.3.3	The Normal MGF	178
3.3.4	The Normal Distribution and Discrete Populations	179
3.3.5	Approximating the Binomial Distribution	180
3.3.6	Normal Distribution Calculations with Software	182
3.3.7	Exercises: Section 3.3 (39–70)	182
3.4	The Exponential and Gamma Distributions	187
3.4.1	The Exponential Distribution	188
3.4.2	The Gamma Distribution	190
3.4.3	The Gamma MGF	193
3.4.4	Gamma and Exponential Calculations with Software	193
3.4.5	Exercises: Section 3.4 (71–83)	194
3.5	Other Continuous Distributions	196
3.5.1	The Weibull Distribution	196
3.5.2	The Lognormal Distribution	199
3.5.3	The Beta Distribution	201
3.5.4	Exercises: Section 3.5 (84–100)	202
3.6	Probability Plots	205
3.6.1	Sample Percentiles	205
3.6.2	A Probability Plot	206
3.6.3	Departures from Normality	209
3.6.4	Beyond Normality	211
3.6.5	Probability Plots in Matlab and R	213
3.6.6	Exercises: Section 3.6 (101–111)	213
3.7	Transformations of a Random Variable	216
3.7.1	Exercises: Section 3.7 (112–128)	220
3.8	Simulation of Continuous Random Variables	221
3.8.1	The Inverse CDF Method	221
3.8.2	The Accept–Reject Method	224
3.8.3	Built-In Simulation Packages for Matlab and R	227
3.8.4	Precision of Simulation Results	227
3.8.5	Exercises: Section 3.8 (129–139)	228
3.9	Supplementary Exercises (140–172)	230
4	Joint Probability Distributions and Their Applications	239
4.1	Jointly Distributed Random Variables	239
4.1.1	The Joint Probability Mass Function for Two Discrete Random Variables	240
4.1.2	The Joint Probability Density Function for Two Continuous Random Variables	241
4.1.3	Independent Random Variables	245
4.1.4	More Than Two Random Variables	246
4.1.5	Exercises: Section 4.1 (1–22)	249
4.2	Expected Values, Covariance, and Correlation	255
4.2.1	Properties of Expected Value	256
4.2.2	Covariance	257
4.2.3	Correlation	259

4.2.4	Correlation Versus Causation	262
4.2.5	Exercises: Section 4.2 (23–42)	262
4.3	Properties of Linear Combinations	264
4.3.1	The PDF of a Sum	268
4.3.2	Moment Generating Functions for Linear Combinations	270
4.3.3	Exercises: Section 4.3 (43–65)	272
4.4	Conditional Distributions and Conditional Expectation	277
4.4.1	Conditional Distributions and Independence	279
4.4.2	Conditional Expectation and Variance	280
4.4.3	The Laws of Total Expectation and Variance	281
4.4.4	Exercises: Section 4.4 (66–84)	286
4.5	Limit Theorems (What Happens as n Gets Large)	290
4.5.1	Random Samples	290
4.5.2	The Central Limit Theorem	293
4.5.3	Other Applications of the Central Limit Theorem	297
4.5.4	The Law of Large Numbers	299
4.5.5	Exercises: Section 4.5 (85–102)	300
4.6	Transformations of Jointly Distributed Random Variables	302
4.6.1	The Joint Distribution of Two New Random Variables	303
4.6.2	The Joint Distribution of More Than Two New Variables	306
4.6.3	Exercises: Section 4.6 (103–110)	307
4.7	The Bivariate Normal Distribution	309
4.7.1	Conditional Distributions of X and Y	311
4.7.2	Regression to the Mean	312
4.7.3	The Multivariate Normal Distribution	312
4.7.4	Bivariate Normal Calculations with Software	313
4.7.5	Exercises: Section 4.7 (111–120)	313
4.8	Reliability	315
4.8.1	The Reliability Function	315
4.8.2	Series and Parallel Designs	317
4.8.3	Mean Time to Failure	320
4.8.4	Hazard Functions	321
4.8.5	Exercises: Section 4.8 (121–132)	323
4.9	Order Statistics	326
4.9.1	The Distributions of Y_n and Y_1	326
4.9.2	The Distribution of the i th Order Statistic	328
4.9.3	The Joint Distribution of the n Order Statistics	329
4.9.4	Exercises: Section 4.9 (133–142)	331
4.10	Simulation of Joint Probability Distributions and System Reliability	332
4.10.1	Simulating Values from a Joint PMF	332
4.10.2	Simulating Values from a Joint PDF	334
4.10.3	Simulating a Bivariate Normal Distribution	336
4.10.4	Simulation Methods for Reliability	338
4.10.5	Exercises: Section 4.10 (143–153)	340
4.11	Supplementary Exercises (154–192)	342
5	The Basics of Statistical Inference	351
5.1	Point Estimation	352
5.1.1	Estimates and Estimators	354
5.1.2	Assessing Estimators: Accuracy and Precision	357
5.1.3	Exercises: Section 5.1 (1–23)	360

5.2	Maximum Likelihood Estimation	366
5.2.1	Some Properties of MLEs	372
5.2.2	Exercises: Section 5.2 (24–36)	373
5.3	Confidence Intervals for a Population Mean	375
5.3.1	A Confidence Interval for a Normal Population Mean	376
5.3.2	A Large-Sample Confidence Interval for μ	380
5.3.3	Software for Confidence Interval Calculation	381
5.3.4	Exercises: Section 5.3 (37–50)	382
5.4	Testing Hypotheses About a Population Mean	386
5.4.1	Hypotheses and Test Procedures	386
5.4.2	Test Procedures for Hypotheses About a Population Mean μ	388
5.4.3	<i>P</i> -Values and the One-Sample <i>t</i> Test	389
5.4.4	Errors in Hypothesis Testing and the Power of a Test	392
5.4.5	Software for Hypothesis Test Calculation	395
5.4.6	Exercises: Section 5.4 (51–76)	396
5.5	Inferences for a Population Proportion	401
5.5.1	Confidence Intervals for p	401
5.5.2	Hypothesis Testing for p	403
5.5.3	Software for Inferences about p	405
5.5.4	Exercises: Section 5.5 (77–97)	405
5.6	Bayesian Inference	409
5.6.1	The Posterior Distribution of a Parameter	410
5.6.2	Inferences from the Posterior Distribution	413
5.6.3	Further Comments on Bayesian Inference	413
5.6.4	Exercises: Section 5.6 (98–106)	414
5.7	Supplementary Exercises (107–138)	416
6	Markov Chains	423
6.1	Terminology and Basic Properties	423
6.1.1	The Markov Property	426
6.1.2	Exercises: Section 6.1 (1–10)	428
6.2	The Transition Matrix and the Chapman–Kolmogorov Equations	431
6.2.1	The Transition Matrix	431
6.2.2	Computation of Multistep Transition Probabilities	432
6.2.3	Exercises: Section 6.2 (11–22)	436
6.3	Specifying an Initial Distribution	440
6.3.1	A Fixed Initial State	443
6.3.2	Exercises: Section 6.3 (23–30)	444
6.4	Regular Markov Chains and the Steady-State Theorem	446
6.4.1	Regular Chains	446
6.4.2	The Steady-State Theorem	448
6.4.3	Interpreting the Steady-State Distribution	450
6.4.4	Efficient Computation of Steady-State Probabilities	451
6.4.5	Irreducible and Periodic Chains	453
6.4.6	Exercises: Section 6.4 (31–43)	454
6.5	Markov Chains with Absorbing States	457
6.5.1	Time to Absorption	458
6.5.2	Mean Time to Absorption	461
6.5.3	Mean First Passage Times	465
6.5.4	Probabilities of Eventual Absorption	466
6.5.5	Exercises: Section 6.5 (44–58)	469

6.6	Simulation of Markov chains	472
6.6.1	Exercises: Section 6.6 (59–66)	479
6.7	Supplementary Exercises (67–82)	481
7	Random Processes	489
7.1	Types of Random Processes	489
7.1.1	Classification of Processes	493
7.1.2	Random Processes Regarded as Random Variables	493
7.1.3	Exercises: Section 7.1 (1–10)	494
7.2	Properties of the Ensemble: Mean and Autocorrelation Functions	496
7.2.1	Mean and Variance Functions	496
7.2.2	Autocovariance and Autocorrelation Functions	499
7.2.3	The Joint Distribution of Two Random Processes	502
7.2.4	Exercises: Section 7.2 (11–24)	503
7.3	Stationary and Wide-Sense Stationary Processes	504
7.3.1	Properties of Wide-Sense Stationary Processes	508
7.3.2	Ergodic Processes	511
7.3.3	Exercises: Section 7.3 (25–40)	514
7.4	Discrete-Time Random Processes	516
7.4.1	Special Discrete Sequences	518
7.4.2	Exercises: Section 7.4 (41–52)	520
7.5	Poisson Processes	522
7.5.1	Relation to Exponential and Gamma Distributions	524
7.5.2	Combining and Decomposing Poisson Processes	526
7.5.3	Alternative Definition of a Poisson Process	528
7.5.4	Nonhomogeneous Poisson Processes	530
7.5.5	The Poisson Telegraphic Process	531
7.5.6	Exercises: Section 7.5 (53–72)	532
7.6	Gaussian Processes	535
7.6.1	Brownian Motion	536
7.6.2	Brownian Motion as a Limit	538
7.6.3	Further Properties of Brownian Motion	538
7.6.4	Variations on Brownian Motion	541
7.6.5	Exercises: Section 7.6 (73–85)	541
7.7	Continuous-Time Markov Chains	544
7.7.1	Infinitesimal Parameters and Instantaneous Transition Rates	546
7.7.2	Sojourn Times and Transitions	548
7.7.3	Long-Run Behavior of Continuous-Time Markov Chains	552
7.7.4	Explicit Form of the Transition Matrix	554
7.7.5	Exercises: Section 7.7 (86–97)	556
7.8	Supplementary Exercises (98–114)	559
8	Introduction to Signal Processing	563
8.1	Power Spectral Density	563
8.1.1	Properties of the Power Spectral Density	566
8.1.2	Power in a Frequency Band	569
8.1.3	White Noise Processes	570
8.1.4	Power Spectral Density for Two Processes	572
8.1.5	Exercises: Section 8.1 (1–21)	573
8.2	Random Processes and LTI Systems	576
8.2.1	Statistical Properties of the LTI System Output	577

8.2.2	Ideal Filters	580
8.2.3	Signal Plus Noise	583
8.2.4	Exercises: Section 8.2 (22–38)	586
8.3	Discrete-Time Signal Processing	589
8.3.1	Random Sequences and LTI Systems	591
8.3.2	Random Sequences and Sampling	593
8.3.3	Exercises: Section 8.3 (39–50)	595
Appendix A: Statistical Tables		597
Appendix B: Background Mathematics		609
Appendix C: Important Probability Distributions		615
Answers to Odd-Numbered Exercises		621
References		637
Index		639