

# Contents

## Part I Newtonian Mechanics in Moving Coordinate Systems

<b>1</b>	<b>Newton's Equations in a Rotating Coordinate System</b>	3
1.1	Introduction of the Operator $\widehat{D}$	6
1.2	Formulation of Newton's Equation in the Rotating Coordinate System	7
1.3	Newton's Equations in Systems with Arbitrary Relative Motion	7
<b>2</b>	<b>Free Fall on the Rotating Earth</b>	9
2.1	Perturbation Calculation	11
2.2	Method of Successive Approximation	12
2.3	Exact Solution	14
<b>3</b>	<b>Foucault's Pendulum</b>	23
3.1	Solution of the Differential Equations	26
3.2	Discussion of the Solution	28

## Part II Mechanics of Particle Systems

<b>4</b>	<b>Degrees of Freedom</b>	41
4.1	Degrees of Freedom of a Rigid Body	41
<b>5</b>	<b>Center of Gravity</b>	43
<b>6</b>	<b>Mechanical Fundamental Quantities of Systems of Mass Points</b>	65
6.1	Linear Momentum of the Many-Body System	65
6.2	Angular Momentum of the Many-Body System	65
6.3	Energy Law of the Many-Body System	68
6.4	Transformation to Center-of-Mass Coordinates	70
6.5	Transformation of the Kinetic Energy	72

## Part III Vibrating Systems

<b>7</b>	<b>Vibrations of Coupled Mass Points</b>	81
7.1	The Vibrating Chain	88
<b>8</b>	<b>The Vibrating String</b>	101
8.1	Solution of the Wave Equation	103
8.2	Normal Vibrations	105
<b>9</b>	<b>Fourier Series</b>	121

<b>10 The Vibrating Membrane . . . . .</b>	133
10.1 Derivation of the Differential Equation . . . . .	133
10.2 Solution of the Differential Equation . . . . .	135
10.3 Inclusion of the Boundary Conditions . . . . .	136
10.4 Eigenfrequencies . . . . .	137
10.5 Degeneracy . . . . .	137
10.6 Nodal Lines . . . . .	138
10.7 General Solution . . . . .	138
10.8 Superposition of Node Line Figures . . . . .	140
10.9 The Circular Membrane . . . . .	141
10.10 Solution of Bessel's Differential Equation . . . . .	144
 <b>Part IV Mechanics of Rigid Bodies</b>	
<b>11 Rotation About a Fixed Axis . . . . .</b>	161
11.1 Moment of Inertia . . . . .	162
11.2 The Physical Pendulum . . . . .	166
<b>12 Rotation About a Point . . . . .</b>	185
12.1 Tensor of Inertia . . . . .	185
12.2 Kinetic Energy of a Rotating Rigid Body . . . . .	187
12.3 The Principal Axes of Inertia . . . . .	188
12.4 Existence and Orthogonality of the Principal Axes . . . . .	189
12.5 Transformation of the Tensor of Inertia . . . . .	193
12.6 Tensor of Inertia in the System of Principal Axes . . . . .	195
12.7 Ellipsoid of Inertia . . . . .	196
<b>13 Theory of the Top . . . . .</b>	209
13.1 The Free Top . . . . .	209
13.2 Geometrical Theory of the Top . . . . .	210
13.3 Analytical Theory of the Free Top . . . . .	213
13.4 The Heavy Symmetric Top: Elementary Considerations . . . . .	224
13.5 Further Applications of the Top . . . . .	228
13.6 The Euler Angles . . . . .	238
13.7 Motion of the Heavy Symmetric Top . . . . .	241
 <b>Part V Lagrange Equations</b>	
<b>14 Generalized Coordinates . . . . .</b>	259
14.1 Quantities of Mechanics in Generalized Coordinates . . . . .	264
<b>15 D'Alembert Principle and Derivation of the Lagrange Equations . . . . .</b>	267
15.1 Virtual Displacements . . . . .	267
<b>16 Lagrange Equation for Nonholonomic Constraints . . . . .</b>	301
<b>17 Special Problems . . . . .</b>	311
17.1 Velocity-Dependent Potentials . . . . .	311
17.2 Nonconservative Forces and Dissipation Function (Friction Function) .	315
17.3 Nonholonomic Systems and Lagrange Multipliers . . . . .	317

**Part VI Hamiltonian Theory**

<b>18 Hamilton's Equations</b> . . . . .	327
18.1 The Hamilton Principle . . . . .	337
18.2 General Discussion of Variational Principles . . . . .	340
18.3 Phase Space and Liouville's Theorem . . . . .	350
18.4 The Principle of Stochastic Cooling . . . . .	355

<b>19 Canonical Transformations</b> . . . . .	365
---	-----

<b>20 Hamilton–Jacobi Theory</b> . . . . .	383
20.1 Visual Interpretation of the Action Function $S$ . . . . .	397
20.2 Transition to Quantum Mechanics . . . . .	407

<b>21 Extended Hamilton–Lagrange Formalism</b> . . . . .	415
21.1 Extended Set of Euler–Lagrange Equations . . . . .	415
21.2 Extended Set of Canonical Equations . . . . .	419
21.3 Extended Canonical Transformations . . . . .	428

<b>22 Extended Hamilton–Jacobi Equation</b> . . . . .	455
---	-----

**Part VII Nonlinear Dynamics**

<b>23 Dynamical Systems</b> . . . . .	463
23.1 Dissipative Systems: Contraction of the Phase-Space Volume . . . . .	465
23.2 Attractors . . . . .	467
23.3 Equilibrium Solutions . . . . .	469
23.4 Limit Cycles . . . . .	475

<b>24 Stability of Time-Dependent Paths</b> . . . . .	485
24.1 Periodic Solutions . . . . .	486
24.2 Discretization and Poincaré Cuts . . . . .	487

<b>25 Bifurcations</b> . . . . .	495
25.1 Static Bifurcations . . . . .	495
25.2 Bifurcations of Time-Dependent Solutions . . . . .	499

<b>26 Lyapunov Exponents and Chaos</b> . . . . .	503
26.1 One-Dimensional Systems . . . . .	503
26.2 Multidimensional Systems . . . . .	505
26.3 Stretching and Folding in Phase Space . . . . .	508
26.4 Fractal Geometry . . . . .	509

<b>27 Systems with Chaotic Dynamics</b> . . . . .	517
27.1 Dynamics of Discrete Systems . . . . .	517
27.2 One-Dimensional Mappings . . . . .	518

**Part VIII On the History of Mechanics**

<b>28 Emergence of Occidental Physics in the Seventeenth Century</b> . . . . .	555
Notes . . . . .	561
Recommendations for Further Reading on Theoretical Mechanics .	573

<b>Index</b> . . . . .	575
------------------------	-----

## Contents of Examples and Exercises

1.1	Angular Velocity Vector $\omega$ . . . . .	6
1.2	Position Vector $\mathbf{r}$ . . . . .	6
2.1	Eastward Deflection of a Falling Body . . . . .	16
2.2	Eastward Deflection of a Thrown Body . . . . .	17
2.3	Superelevation of a River Bank . . . . .	18
2.4	Difference of Sea Depth at the Pole and Equator . . . . .	18
3.1	Chain Fixed to a Rotating Bar . . . . .	29
3.2	Pendulum in a Moving Train . . . . .	30
3.3	Formation of Cyclones . . . . .	34
3.4	Movable Mass in a Rotating Tube . . . . .	35
5.1	Center of Gravity for a System of Three Mass Points . . . . .	44
5.2	Center of Gravity of a Pyramid . . . . .	45
5.3	Center of Gravity of a Semicircle . . . . .	46
5.4	Center of Gravity of a Circular Cone . . . . .	47
5.5	Momentary Center and Pole Path . . . . .	48
5.6	Scattering in a Central Field . . . . .	50
5.7	Rutherford Scattering Cross Section . . . . .	55
5.8	Scattering of a Particle by a Spherical Square Well Potential . . . . .	59
5.9	Scattering of Two Atoms . . . . .	63
6.1	Conservation of the Total Angular Momentum of a Many-Body System: Flattening of a Galaxy . . . . .	67
6.2	Conservation of Angular Momentum of a Many-Body Problem: The Pirouette . . . . .	67
6.3	Reduced Mass . . . . .	72
6.4	Movement of Two Bodies Under the Action of Mutual Gravitation . . . . .	73
6.5	Atwoods Fall Machine . . . . .	75
6.6	Our Solar System in the Milky Way . . . . .	76
7.1	Two Equal Masses Coupled by Two Equal Springs . . . . .	84
7.2	Coupled Pendulums . . . . .	85
7.3	Eigenfrequencies of the Vibrating Chain . . . . .	94
7.4	Vibration of Two Coupled Mass Points, Two Dimensional . . . . .	96
7.5	Three Masses on a String . . . . .	97
7.6	Eigen vibrations of a Three-Atom Molecule . . . . .	99
8.1	Kinetic and Potential Energy of a Vibrating String . . . . .	108
8.2	Three Different Masses Equidistantly Fixed on a String . . . . .	109

8.3	Complicated Coupled Vibrational System . . . . .	112
8.4	The Cardano Formula . . . . .	113
9.1	Inclusion of the Initial Conditions for the Vibrating String by Means of the Fourier Expansion . . . . .	123
9.2	Fourier Series of the Sawtooth Function . . . . .	125
9.3	Vibrating String with a Given Velocity Distribution . . . . .	126
9.4	Fourier Series for a Step Function . . . . .	128
9.5	On the Unambiguousness of the Tautochrone Problem . . . . .	129
10.1	The Longitudinal Chain: Poincaré Recurrence Time . . . . .	150
10.2	Orthogonality of the Eigenmodes . . . . .	155
11.1	Moment of Inertia of a Homogeneous Circular Cylinder . . . . .	163
11.2	Moment of Inertia of a Thin Rectangular Disk . . . . .	165
11.3	Moment of Inertia of a Sphere . . . . .	167
11.4	Moment of Inertia of a Cube . . . . .	167
11.5	Vibrations of a Suspended Cube . . . . .	168
11.6	Roll off of a Cylinder: Rolling Pendulum . . . . .	169
11.7	Moments of Inertia of Several Rigid Bodies About Selected Axes . . . . .	174
11.8	Cube Tilts over the Edge of a Table . . . . .	175
11.9	Hockey Puck Hits a Bar . . . . .	176
11.10	Cue Pushes a Billiard Ball . . . . .	178
11.11	Motion with Constraints . . . . .	180
11.12	Bar Vibrates on Springs . . . . .	181
12.1	Tensor of Inertia of a Square Covered with Mass . . . . .	191
12.2	Transformation of the Tensor of Inertia of a Square Covered with Mass . . . . .	199
12.3	Rolling Circular Top . . . . .	199
12.4	Ellipsoid of Inertia of a Quadratic Disk . . . . .	202
12.5	Symmetry Axis as a Principal Axis . . . . .	203
12.6	Tensor of Inertia and Ellipsoid of Inertia of a System of Three Masses . . . . .	204
12.7	Friction Forces and Acceleration of a Car . . . . .	206
13.1	Nutation of the Earth . . . . .	217
13.2	Ellipsoid of Inertia of a Regular Polyhedron . . . . .	218
13.3	Rotating Ellipsoid . . . . .	218
13.4	Torque of a Rotating Plate . . . . .	219
13.5	Rotation of a Vibrating Neutron Star . . . . .	220
13.6	Pivot Forces of a Rotating Circular Disk . . . . .	222
13.7	Torque on an Elliptic Disk . . . . .	223
13.8	Gyrocompass . . . . .	229
13.9	Tidal Forces, and Lunar and Solar Eclipses: The Saros Cycle . . . . .	230
13.10	The Sleeping Top . . . . .	247
13.11	The Heavy Symmetric Top . . . . .	249
13.12	Stable and Unstable Rotations of the Asymmetric Top . . . . .	255
14.1	Small Sphere Rolls on a Large Sphere . . . . .	260
14.2	Body Glides on an Inclined Plane . . . . .	260
14.3	Wheel Rolls on a Plane . . . . .	261
14.4	Generalized Coordinates . . . . .	262
14.5	Cylinder Rolls on an Inclined Plane . . . . .	263

14.6	Classification of Constraints . . . . .	263
15.1	Two Masses on Concentric Rollers . . . . .	270
15.2	Two Masses Connected by a Rope on an Inclined Plane . . . . .	271
15.3	Equilibrium Condition of a Bascule Bridge . . . . .	271
15.4	Two Blocks Connected by a Bar . . . . .	276
15.5	Ignorable Coordinate . . . . .	277
15.6	Sphere in a Rotating Tube . . . . .	279
15.7	Upright Pendulum . . . . .	281
15.8	Stable Equilibrium Position of an Upright Pendulum . . . . .	282
15.9	Vibration Frequencies of a Three-Atom Symmetric Molecule . . . . .	284
15.10	Normal Frequencies of a Triangular Molecule . . . . .	286
15.11	Normal Frequencies of an Asymmetric Linear Molecule . . . . .	289
15.12	Double Pendulum . . . . .	289
15.13	Mass Point on a Cycloid Trajectory . . . . .	292
15.14	String Pendulum . . . . .	294
15.15	Coupled Mass Points on a Circle . . . . .	295
15.16	Lagrangian of the Asymmetric Top . . . . .	297
16.1	Cylinder Rolls down an Inclined Plane . . . . .	303
16.2	Particle Moves in a Paraboloid . . . . .	305
16.3	Three Masses Coupled by Rods Glide in a Circular Tire . . . . .	308
17.1	Charged Particle in an Electromagnetic Field . . . . .	314
17.2	Motion of a Projectile in Air . . . . .	317
17.3	Circular Disk Rolls on a Plane . . . . .	320
17.4	Centrifugal Force Governor . . . . .	322
18.1	Central Motion . . . . .	331
18.2	The Pendulum in the Newtonian, Lagrangian, and Hamiltonian Theories	332
18.3	Hamiltonian and Canonical Equations of Motion . . . . .	334
18.4	A Variational Problem . . . . .	338
18.5	Catenary . . . . .	342
18.6	Brachistochrone: Construction of an Emergency Chute . . . . .	344
18.7	Derivation of the Hamiltonian Equations . . . . .	349
18.8	Phase Diagram of a Plane Pendulum . . . . .	351
18.9	Phase-Space Density for Particles in the Gravitational Field . . . . .	354
18.10	Cooling of a Particle Beam . . . . .	359
19.1	Example of a Canonical Transformation . . . . .	370
19.2	Point Transformations . . . . .	370
19.3	Harmonic Oscillator . . . . .	370
19.4	Damped Harmonic Oscillator . . . . .	373
19.5	Infinitesimal Time Step . . . . .	375
19.6	General Form of Liouville's Theorem . . . . .	376
19.7	Canonical Invariance of the Poisson Brackets . . . . .	377
19.8	Poisson's Theorem . . . . .	379
19.9	Invariants of the Plane Kepler System . . . . .	380
20.1	The Hamilton–Jacobi Differential Equation . . . . .	385
20.2	Angle Variable . . . . .	389
20.3	Solution of the Kepler Problem by the Hamilton–Jacobi Method . . . . .	389

20.4	Formulation of the Hamilton–Jacobi Differential Equation for Particle Motion in a Potential with Azimuthal Symmetry . . . . .	392
20.5	Solution of the Hamilton–Jacobi Differential Equation of Exercise 20.4 . . . . .	393
20.6	Formulation of the Hamilton–Jacobi Differential Equation for the Slant Throw . . . . .	395
20.7	Illustration of the Action Waves . . . . .	398
20.8	Periodic and Multiply Periodic Motions . . . . .	400
20.9	The Bohr–Sommerfeld Hydrogen Atom . . . . .	408
20.10	On Poisson Brackets . . . . .	410
20.11	Total Time Derivative of an Arbitrary Function Depending on $q$ , $p$ , and $t$ . . . . .	412
21.1	Extended Lagrangian for a Relativistic Free Particle . . . . .	417
21.2	Extended Lagrangian for a Relativistic Particle in an External Electromagnetic Field . . . . .	418
21.3	Trivial Extended Hamiltonian . . . . .	421
21.4	Hamiltonian of a Free Relativistic Particle . . . . .	422
21.5	Hamiltonian of a Relativistic Particle in a Potential $V(\mathbf{q}, t)$ . . . . .	423
21.6	Relativistic “Harmonic Oscillator” . . . . .	425
21.7	Extended Hamiltonian for a Relativistic Particle in an External Electromagnetic Field . . . . .	426
21.8	Identical Canonical Transformation . . . . .	432
21.9	Identical Time Transformation, Conventional Canonical Transformations . . . . .	432
21.10	Extended Point Transformations . . . . .	433
21.11	Time-Energy Transformations . . . . .	433
21.12	Liouville’s Theorem in the Extended Hamilton Description . . . . .	434
21.13	Extended Poisson Brackets . . . . .	434
21.14	Canonical Quantization in the Extended Hamilton Formalism . . . . .	435
21.15	Regularization of the Kepler System . . . . .	437
21.16	Time-Dependent Damped Harmonic Oscillator . . . . .	440
21.17	Galilei Transformation . . . . .	444
21.18	Lorentz Transformation . . . . .	445
21.19	Infinitesimal Canonical Transformations, Generalized Noether Theorem . . . . .	447
21.20	Infinitesimal Point Transformations, Conventional Noether Theorem . . . . .	450
21.21	Runge–Lenz Vector of the Plane Kepler System as a Generalized Noether Invariant . . . . .	451
22.1	Time Dependent Harmonic Oscillator . . . . .	456
23.1	Linear Stability in Two Dimensions . . . . .	471
23.2	The Nonlinear Oscillator with Friction . . . . .	473
23.3	The van der Pol Oscillator with Weak Nonlinearity . . . . .	480
23.4	Relaxation Vibrations . . . . .	482
24.1	Floquet’s Theory of Stability . . . . .	489
24.2	Stability of a Limit Cycle . . . . .	492
26.1	The Baker Transformation . . . . .	515
27.1	The Logistic Mapping . . . . .	519
27.2	Logistic Mapping and the Bernoulli Shift . . . . .	527
27.3	The Periodically Kicked Rotator . . . . .	530
27.4	The Periodically Driven Pendulum . . . . .	537
27.5	Chaos in Celestial Mechanics: The Staggering of Hyperion . . . . .	544