

Contents

Preface	vii
Abbreviated Contents for Volume II	xx
1 Introduction	1
1.0 Historical Perspective	1
1.1 Heterogeneous Materials	2
1.2 Effective Properties of Heterogeneous Materials	4
1.3 Linear Transport Properties	5
1.3.1 The Effective Conductivity	5
1.3.2 The Effective Dielectric Constant	6
1.3.3 The Effective Elastic Moduli	6
1.4 Nonlinear Transport Properties	7
1.4.1 Constitutive Nonlinearity	7
1.4.2 Threshold Nonlinearity	8
1.5 Predicting the Effective Properties of Heterogeneous Materials	8
1.5.1 The Continuum Models	9
1.5.2 The Discrete Models	10
1.6 The Organization of the Book	11
I Characterization and Modeling of the Morphology	13
2 Characterization of Connectivity and Clustering	15
2.0 Introduction	15
2.1 Characterization of the Geometry: Self-Similar Fractal Microstructures	17
2.2 Statistical Self-Similarity	19
2.3 Measurement of the Fractal Dimension	20
2.3.1 The Correlation Function Method	21
2.3.2 Small-Angle Scattering	21
2.4 Self-Affine Fractals	24
2.5 Characterization of Connectivity and Clustering	26
2.5.1 Random Bond and Site Percolation	27
2.5.2 Percolation Thresholds	29

2.5.3	Bicontinuous Materials, Phase-Inversion Symmetry, and Percolation	30
2.5.4	Computer Generation of a Single Cluster	30
2.6	Percolation Properties	31
2.6.1	Morphological Properties	32
2.6.2	Transport Properties	32
2.6.3	The Structure of the Sample-Spanning Cluster	34
2.7	Universal Scaling Properties of Percolation	35
2.7.1	Morphological Properties	35
2.7.2	Transport Properties	36
2.7.3	Practical Significance of the Critical Exponents	37
2.8	Scale-Dependent Properties of Percolation Composites	38
2.9	Finite-Size Scaling	40
2.10	Percolation in Random Networks and Continua	41
2.10.1	Percolation Thresholds: Materials with Very Low or High Thresholds	42
2.10.2	The Ornstein-Zernike Formulation	44
2.11	Differences Between Lattice and Continuum Percolation	45
2.12	Correlated Percolation	48
2.12.1	Short-Range Correlations	49
2.12.2	Long-Range Correlations	50
2.13	Experimental Measurement of Percolation Properties	52
	Summary	56
3	Characterization and Modeling of the Morphology	57
3.0	Introduction	57
3.1	Models of Heterogeneous Materials	58
3.2	One-Dimensional Models	59
3.3	Spatially Periodic Models	61
3.4	Continuum Models	62
3.4.1	Dispersion of Spheres	63
3.4.1.1	Equilibrium Hard-Sphere Model	65
3.4.1.2	Random Close Packing Versus Maximally Random Jamming	66
3.4.1.3	Particle Distribution and Correlation Functions	66
3.4.1.4	The n -Particle Probability Density	73
3.4.2	Distribution of Equal-Size Particles	75
3.4.2.1	Fully Penetrable Spheres	75
3.4.2.2	Fully Impenetrable Spheres	77
3.4.2.3	Interpenetrable Spheres	80
3.4.3	Distribution of Polydispersed Spheres	80
3.4.3.1	Fully Penetrable Spheres	81
3.4.3.2	Fully Impenetrable Spheres	82
3.4.4	Simulation of Dispersion of Spheres	83
3.4.5	Models of Anisotropic Materials	84

3.4.6	Tessellation Models of Cellular Materials	85
3.4.7	Gaussian Random Field Models of Amorphous Materials	91
3.5	Discrete Models	95
3.5.1	Network Models	95
3.5.2	Bethe Lattice Models	96
3.6	Reconstruction of Heterogeneous Materials: Simulated Annealing	97
	Summary	101

II Linear Transport and Optical Properties 103

4	Effective Conductivity, Dielectric Constant, and Optical Properties: The Continuum Approach	105
4.0	Introduction	105
4.1	Symmetry Properties of the Conductivity Tensor	108
4.2	General Results	108
4.3	Effective Conductivity of Dispersion of Spheres: Exact Results	110
4.3.1	Three-Dimensional Regular Arrays of Spheres	111
4.3.1.1	Simple-Cubic arrays	111
4.3.1.2	Body-Centered and Face-Centered Cubic Arrays	118
4.3.2	Two-Dimensional Arrays of Cylinders	120
4.3.2.1	Hexagonal Arrays	121
4.3.2.2	Square Arrays	123
4.4	Exact Results for Coated Spheres and Laminates	125
4.5	Perturbation Expansion for the Effective Conductivity	127
4.5.1	Isotropic Materials: Strong-Contrast Expansion	128
4.5.2	Approximations	132
4.5.3	The Microstructural Parameter ζ_2	133
4.5.4	Anisotropic Materials	136
4.6	Bounds on Effective Conductivity	138
4.6.1	Isotropic Materials	139
4.6.1.1	Two-Point Bounds	139
4.6.1.2	Cluster Bounds	141
4.6.1.3	Three- and Four-Point Bounds	146
4.6.1.4	Cluster Expansions for the Effective Conductivity	151
4.6.2	Anisotropic Materials	152
4.6.2.1	Two-Point Bounds	152
4.6.2.2	Three- and Four-Point Bounds	153
4.6.2.3	Simplification of the Bounds	154
4.6.2.4	Cluster Expansions for the Effective Conductivity	158
4.7	The Effect of the Interface on the Effective Conductivity	160

4.8	Exact Duality Relations	161	5.6.3	Resistor Networks with Multiple Coordination Numbers	218
4.9	Effective-Medium Approximation	162	5.6.4	Materials with Zero Percolation Threshold	221
	4.9.1 Isotropic Materials	163	5.6.5	Comparison with the Experimental Data	223
	4.9.2 Anisotropic Materials	164	5.6.6	Accuracy of the Effective- Medium Approximation	225
	4.9.3 Critique of the Effective- Medium Approximation	167	5.6.7	Cluster Effective-Medium Approximation	229
	4.9.4 The Maxwell–Garnett Approximation	168	5.6.8	Coherent-Potential Approximation	230
4.10	The Random Walk Method	169	5.7	Effective-Medium Approximation for Site Percolation	231
4.11	The Effective Dielectric Constant	176	5.8	Effective-Medium Approximation for Correlated Composites	234
	4.11.1 Spectral Representation	177	5.9	Effective-Medium Approximation for Anisotropic Materials	236
	4.11.2 Perturbation Expansion	179		5.9.1 The Green Functions	237
	4.11.3 Rigorous Bounds	180		5.9.2 Conductivity Anisotropy Near the Percolation Threshold	240
4.12	Optical Properties	182		5.9.3 Comparison with the Experimental Data	241
	4.12.1 Conductor–Insulator Composites	183	5.10	Cumulant Approximation	242
	4.12.2 Conductor–Superconductor Composites	186		5.10.1 The Lorentz Field	244
	4.12.3 Anisotropic Materials	187		5.10.2 Perturbation Expansion	244
	4.12.4 The Cole–Cole Representation	188		5.10.3 Computation of the Lowest-Order Terms	245
4.13	Beyond the Quasi-static Approximation: Mie Scattering	189		5.10.4 Bond Percolation	247
4.14	Dynamical Effective-Medium Approximation	192	5.11	Position-Space Renormalization Group Methods	248
4.15	The Effect of Large-Scale Morphology	194	5.12	Renormalized Effective-Medium Approximation	253
4.16	Multiple-Scattering Approach	194	5.13	The Critical Path Method	255
	Summary	196	5.14	Numerical Computation of the Effective Conductivity	258
5	Effective Conductivity and Dielectric Constant: The Discrete Approach	197		5.14.1 The Conjugate-Gradient Method	258
5.0	Introduction	197		5.14.2 Transfer-Matrix Method	259
5.1	Experimental Data for Conduction in Heterogeneous Materials	198		5.14.3 Network Reduction: The Lobb–Frank–Fogelholm Methods	261
	5.1.1 Powders	198		5.14.4 Random Walk Method	263
	5.1.2 Polymer Composites	199	5.15	Estimation of the Critical Exponents of the Conductivity	264
	5.1.3 Conductor–Insulator Composites	200		5.15.1 Finite-Size Scaling	264
5.2	Conductivity of a Random Resistor Network	202		5.15.2 Position-Space Renormalization Group Method	265
5.3	Exact Solution for Bethe Lattices	203		5.15.3 Series Expansion	266
	5.3.1 The Microscopic Conductivity	204		5.15.4 Field-Theoretic Approach	267
	5.3.2 Effective-Medium Approximation	206		5.15.5 Comparison with the Experimental Measurements	268
	5.3.3 Conductor–Insulator Composites	206	5.16	Resistance Fluctuations, Moments of the Current Distribution, and Flicker Noise	270
	5.3.4 Conductor–Superconductor Composites	207		5.16.1 Tellegen’s Theorems	271
5.4	Exact Results for Two-Dimensional Composites	207		5.16.2 Cohn’s Theorem	271
	5.4.1 Exact Duality Relations	207		5.16.3 Scaling Properties	272
	5.4.2 Log-Normal Conductance Distribution	209		5.16.4 Comparison with the Experimental Data	274
5.5	Green Function Formulation and Perturbation Expansion	209	5.17	Hall Conductivity	275
	5.5.1 Properties of the Green Functions	211		5.17.1 Effective-Medium Approximation	276
5.6	Effective-Medium Approximation	213		5.17.2 Network Model	279
	5.6.1 Conductor–Insulator Composites	216			
	5.6.2 Conductor–Superconductor Composites	218			

5.17.3	Exact Duality Relations	280	6.2.3	The Asymmetric Hopping Model: Perturbation Expansion	319
5.17.4	Scaling Properties	281	6.2.3.1	Exact Solution for Bethe Lattices	321
5.17.5	Comparison with Experimental Data	282	6.2.3.2	Two-Site Self-Consistent Approximation	322
5.18	Classical Aspects of Superconductivity	284	6.2.3.3	Two-Site Effective- Medium Approximation	323
5.18.1	Magnetoconductivity	284	6.2.3.4	Energy-Dependent Effective- Medium Approximation	324
5.18.2	Magnetic Properties	285	6.2.4	Variable-Range Hopping: The Critical Path Method	325
5.18.3	Comparison with the Experimental Data	286	6.2.4.1	Effect of a Variable Density of States	329
5.18.3.1	The London Penetration Depth	286	6.2.4.2	Effect of Coulomb Interactions	330
5.18.3.2	The Specific Heat	287	6.2.4.3	Comparison with the Experimental Data	330
5.18.3.3	The Critical Current	287	6.2.4.4	Fractal Morphology and Superlocalization	335
5.18.3.4	The Critical Fields	288	6.2.5	Continuous-Time Random Walk Model	337
5.18.3.5	Differential Diamagnetic Susceptibility	289	6.3	AC Conductivity	339
Summary	290	6.3.1	Universality of AC Conductivity	340
6	Frequency-Dependent Properties: The Discrete Approach	291	6.3.2	Resistor-Capacitor Model	345
6.0	Introduction	291	6.3.3	Universal AC Conductivity: Effective- Medium Approximation	346
6.1	Diffusion in Heterogeneous Materials	292	6.3.4	Universal AC Conductivity: Symmetric Hopping Model	349
6.1.1	Green Function Formulation and Perturbation Expansion	293	6.3.5	Role of Percolation in Universality of AC Conductivity	350
6.1.2	Self-Consistent Approach	294	6.4	Dielectric Constant and Optical Properties	352
6.1.3	Self-Consistent Approximation, Generalized Master Equation and Continuous-Time Random Walks	295	6.4.1	Resistor-Capacitor Model	352
6.1.4	The Green Functions	295	6.4.2	Resistor-Capacitor-Inductor Model	354
6.1.5	Effective-Medium Approximation	297	6.4.3	Position-Space Renormalization Group Approach	356
6.1.6	The Mean Square Displacement	298	6.4.4	Effective-Medium Approximation	360
6.1.7	Difference Between Transport in Low- and High-Dimensional Materials	299	6.4.5	Random Walk Model	362
6.1.8	Predictions of the Effective- Medium Approximation	300	6.5	Scaling Properties of AC Conductivity and Dielectric Constant	365
6.1.8.1	One-Dimensional Materials	300	6.5.1	Comparison with the Experimental Data	369
6.1.8.2	Two- and Three- Dimensional Materials	302	6.6	Vibrational Density of States: The Scalar Approximation	375
6.1.9	Anomalous Diffusion	305	6.6.1	Numerical Computation	377
6.1.10	Scaling Theory of Anomalous Diffusion	306	6.6.2	Effective-Medium Approximation	378
6.1.11	Comparison with the Experimental Data	308	6.6.3	Cluster Effective-Medium Approximation	382
6.1.12	The Governing Equation for Anomalous Diffusion	309	6.6.4	Scaling Theory: Phonons Versus Fractons	383
6.2	Hopping Conductivity	309	6.6.5	Characteristics of Fractons	384
6.2.1	The Miller-Abrahams Network Model	311	6.6.5.1	Localization	384
6.2.2	The Symmetric Hopping Model	314	6.6.5.2	Dispersion Relation	385
6.2.2.1	Exact Solution for One-Dimensional Materials	314	6.6.5.3	Crossover from Phonons to Fractons	385
6.2.2.2	Exact Solution for Bethe Lattices	316			
6.2.2.3	Perturbation Expansion and Effective-Medium Approximation	317			

6.6.6	Large-Scale Computer Simulations	386
6.6.7	Missing Modes	389
6.6.8	Localization Properties of Fractons	393
6.6.8.1	Mode Patterns of Fractons	393
6.6.8.2	Ensemble-Averaged Fractons	393
6.6.9	Comparison with the Experimental Data	395
6.7	The Dynamical Structure Factor	398
6.7.1	Theoretical Analysis	399
6.7.2	Scaling Analysis	400
6.7.3	Numerical Computation	402
6.8	Fractons and Thermal Transport in Heterogeneous Materials	404
6.8.1	Anharmonicity	405
6.8.2	Phonon-Assisted Fracton Hopping	405
6.8.3	Dependence of Sound Velocity on Temperature	407
	Summary	409
7	Rigidity and Elastic Properties: The Continuum Approach	410
7.0	Introduction	410
7.1	The Stress and Strain Tensors	411
7.1.1	Symmetry Properties of the Stiffness Tensor	412
7.1.2	Theorems of Minimum Energy	415
7.1.3	The Strain Energy of a Composite Material	416
7.1.4	Volume Averaging	417
7.2	Exact Results	419
7.2.1	Interrelations Between Two- and Three- Dimensional Moduli	420
7.2.2	Exact Results for Regular Arrays of Spheres	421
7.2.3	Exact Results for Coated Spheres and Laminates	422
7.2.4	Connection to Two-Dimensional Conductivity	425
7.2.5	Exact Duality Relations	425
7.2.6	The Cherkaev–Lurie–Milton Theorem and Transformation	426
7.2.7	Universality of Poisson’s Ratio in Percolation Composites	427
7.2.8	Composite Materials with Equal Shear Moduli	429
7.2.9	Dundurs Constants	429
7.2.10	Relations Between Elastic Moduli and Thermoelastic Properties	430
7.3	Dispersion of Spherical Inclusions	431
7.3.1	The Dilute Limit: A Single Sphere	431
7.3.2	Nondilute Dispersions	433
7.3.3	Two Spherical Inclusions	437
7.4	Exact Strong-Contrast Expansions	447
7.4.1	Integral Equation for the Cavity Strain Field	447
7.4.2	Exact Series Expansions	452

7.4.3	Exact Series Expansions for Isotropic Materials	455
7.4.4	Macroscopically Anisotropic Materials	460
7.4.5	The Microstructural Parameter η_2	460
7.4.6	Comparison with Numerical Simulation	461
7.4.6.1	Two-Dimensional materials	461
7.4.6.2	Three-Dimensional materials	466
7.5	Rigorous Bounds	471
7.5.1	Isotropic Materials	472
7.5.1.1	One-Point Bounds	472
7.5.1.2	Two-Point Bounds	472
7.5.1.3	Cluster Bounds	475
7.5.1.4	Three- and Four-Point Bounds	485
7.5.2	Anisotropic Materials	488
7.6	Multiple Scattering Method	493
7.6.1	The Dilute Limit	497
7.6.2	Nondilute Systems	499
7.6.3	Comparison with the Experimental Data	501
7.7	Effective-Medium Approximations	504
7.7.1	Fundamental Tensors and Invariant Properties	505
7.7.2	Symmetric Effective-Medium Approximation	508
7.7.3	Asymmetric Effective-Medium Approximation	510
7.7.4	The Maxwell–Garnett Approximations	513
7.8	Numerical Simulation	514
7.8.1	Finite-Difference Methods	514
7.8.2	Boundary-Element and Finite-Element Methods	514
7.9	Links Between the Conductivity and Elastic Moduli	518
7.9.1	Two-Dimensional Materials	518
7.9.1.1	Conductivity-Bulk Modulus Bounds	520
7.9.1.2	Conductivity-Shear Modulus Bounds	521
7.9.1.3	Applications	525
7.9.2	Three-Dimensional Materials	526
7.9.2.1	Conductivity-Bulk Modulus Bounds	527
7.9.2.2	Applications	530
	Summary	531
8	Rigidity and Elastic Properties: The Discrete Approach	532
8.0	Introduction	532
8.1	Elastic Networks in Biological Materials	534
8.2	Number of Elastic Moduli of a Lattice	536
8.3	Numerical Simulation and Finite-Size Scaling	537
8.4	Derivation of Elastic Networks from Continuum Elasticity	539
8.4.1	The Born Model	541
8.4.2	Shortcomings of the Born Model	541
8.5	The Central-Force Network	543

8.6	Rigidity Percolation	545
8.6.1	Static and Dynamic Rigidity and Floppiness of Networks	546
8.6.2	The Correlation Length of Rigidity Percolation	548
8.6.3	The Force Distribution	549
8.6.4	Determination of the Percolation Threshold	550
8.6.4.1	Moments of the Force Distribution	550
8.6.4.2	The Pebble Game	551
8.6.4.3	Constraint-Counting Method	554
8.6.5	Mapping Between Rigidity Percolation and Resistor Networks	555
8.6.6	Nature of the Phase Transition	556
8.6.7	Scaling Properties of the Elastic Moduli	557
8.7	Green Function Formulation and Perturbation Expansion	560
8.7.1	Effective-Medium Approximation	561
8.7.2	The Born Model	561
8.7.3	Rigidity Percolation	563
8.8	The Critical Path Method	565
8.9	Central-Force Networks at Nonzero Temperatures and Under Stress	565
8.10	Shortcomings of the Central-Force Networks	568
8.11	Elastic Percolation Networks with Bond-Bending Forces	569
8.11.1	The Kirkwood-Keating Model	570
8.11.2	The Bond-Bending Model	570
8.11.3	The Percolation Thresholds	571
8.11.4	The Force Distribution	572
8.11.5	Comparison of the Central-Force and Bond-Bending Networks	573
8.11.6	Scaling Properties	574
8.11.7	Relation with Scalar Percolation	576
8.11.8	Fixed Points of Vector Percolation: Universality of the Poisson's Ratio	577
8.11.9	Position-Space Renormalization Group Method	578
8.11.10	Effective-Medium Approximation	581
8.12	Transfer-Matrix Method	581
8.13	The Beam Model	582
8.14	The Granular Model	583
8.15	Entropic Networks	584
	Summary	585
9	Rigidity and Elastic Properties of Network Glasses, Polymers, and Composite Solids: The Discrete Approach	587
9.0	Introduction	587
9.1	Network Glasses	589
9.1.1	Rigidity Transition	591
9.1.2	Comparison with the Experimental Data	592

9.1.3	Rigidity Transition at High Coordination Numbers	595
9.1.4	Effect of Onefold-Coordinated Atoms	597
9.1.5	Stress-Free Versus Stressed Transition	598
9.2	Branched Polymers and Gels	599
9.2.1	Percolation Model of Polymerization and Gelation	602
9.2.2	Morphological Properties of Branched Polymers and Gels	602
9.2.2.1	Gel Polymers	603
9.2.2.2	Comparison with the Experimental Data	604
9.2.2.3	Branched Polymers	606
9.2.2.4	Comparison with the Experimental Data	608
9.2.3	Rheology of Critical Gels: Dynamic-Mechanical Experiments	609
9.2.4	The Relaxation Time Spectrum	611
9.2.5	Comparison with the Experimental Data	614
9.2.5.1	Physical Gels	614
9.2.5.2	Chemical Gels	614
9.2.5.3	Enthalpic Versus Entropic Elasticity	616
9.2.5.4	Viscosity of Near-Critical Gelling Solutions	618
9.3	Mechanical Properties of Foams	621
9.4	Mechanical Properties of Composite Solids	624
9.4.1	Porous Materials	624
9.4.2	Superrigid Materials	627
9.5	Wave Speeds in Porous Materials	627
9.6	Elastic Properties of Composite Materials with Length Mismatch	628
9.7	Materials with Negative Poisson's Ratio	631
9.8	Vibrational Density of States: Vector Percolation Model	634
9.8.1	Scaling Theory	634
9.8.2	Crossover Between Scalar Approximation and Vector Density of States	637
9.8.3	Large-Scale Computer Simulation	637
9.8.4	Comparison with the Experimental Data	640
	Summary	642

References	643
-----------------------------	------------

Index	685
------------------------	------------