

Contents

I. *The one-dimensional wave equation*

1. A physical problem and its mathematical models: the vibrating string	1
2. The one-dimensional wave equation	8
3. Discussion of the solution: characteristics	18
4. Reflection and the free boundary problem	21
5. The nonhomogeneous wave equation	24

II. *Linear second-order partial differential equations in two variables*

6. Linearity and superposition	29
7. Uniqueness for the vibrating string problem	36
8. Classification of second-order equations with constant coefficients	41
9. Classification of general second-order operators	44

III. *Some properties of elliptic and parabolic equations*

10. Laplace's equation	48
11. Green's theorem and uniqueness for the Laplace's equation	52
12. The maximum principle	55
13. The heat equation	58

IV. *Separation of variables and Fourier series*

14. The method of separation of variables	63
15. Orthogonality and least square approximation	70
16. Completeness and the Parseval equation	73
17. The Riemann-Lebesgue lemma	76
18. Convergence of the trigonometric Fourier series	77
19. Uniform convergence, Schwarz's inequality, and completeness	81
20. Sine and cosine series	87
21. Change of scale	88
22. The heat equation	92
23. Laplace's equation in a rectangle	95
24. Laplace's equation in a circle	100
25. An extension of the validity of these solutions	105
26. The damped wave equation	112

V. Nonhomogeneous problems	
27. Initial value problems for ordinary differential equations	117
28. Boundary value problems and Green's function for ordinary differential equations	
29. Nonhomogeneous problems and the finite Fourier transform	120
30. Green's function	126
	132
VI. Problems in higher dimensions and multiple Fourier series	
31. Multiple Fourier series	141
32. Laplace's equation in a cube	146
33. Laplace's equation in a cylinder	149
34. The three-dimensional wave equation in a cube	152
35. Poisson's equation in a cube	155
VII. Sturm-Liouville theory and general Fourier expansions	
36. Eigenfunction expansions for regular second-order ordinary differential equations	160
37. Vibration of a variable string	169
38. Some properties of eigenvalues and eigenfunctions	171
39. Equations with singular endpoints	176
40. Some properties of Bessel functions	179
41. Vibration of a circular membrane	182
42. Forced vibration of a circular membrane: natural frequencies and resonance	185
43. The Legendre polynomials and associated Legendre functions	188
44. Laplace's equation in the sphere	194
45. Poisson's equation and Green's function for the sphere	197
VIII. Analytic functions of a complex variable	
46. Complex numbers	201
47. Complex power series and harmonic functions	207
48. Analytic functions	213
49. Contour integrals and Cauchy's theorem	218
50. Composition of analytic functions	225
51. Taylor series of composite functions	231
52. Conformal mapping and Laplace's equation	236
53. The bilinear transformation	246
54. Laplace's equation on unbounded domains	253
55. Some special conformal mappings	257
56. The Cauchy integral representation and Liouville's theorem	261
IX. Evaluation of integrals by complex variable methods	
57. Singularities of analytic functions	269
58. The calculus of residues	271

59. Laurent's series	278
60. Infinite integrals	282
61. Infinite series of residues	289
62. Integrals along branch cuts	293
X. The Fourier transform	
63. The Fourier transform	298
64. Jordan's lemma	302
65. Schwarz's inequality and the triangle inequality for infinite integrals	305
66. Fourier transforms of square integrable functions: the Parseval equation	310
67. Fourier inversion theorems	313
68. Sine and cosine transforms	320
69. Some operational formulas	324
70. The convolution product	326
71. Multiple Fourier transforms: the heat equation in three dimensions	329
72. The three-dimensional wave equation	333
73. The Fourier transform with complex argument	337
XI. The Laplace transform	
74. The Laplace transform	346
75. Initial value problems for ordinary differential equations	351
76. Initial value problems for the one-dimensional heat equation	355
77. A diffraction problem	362
78. The Stokes rule and Duhamel's principle	370
XII. Approximation methods	
79. "Exact" and approximate solutions	374
80. The method of finite differences for initial-boundary value problems	375
81. The finite difference method for Laplace's equation	380
82. The method of successive approximations	384
83. The Rayleigh-Ritz method	392
SOLUTIONS TO THE EXERCISES	443
INDEX	399