

Contents

List of figures	xv
1 What is statistical mechanics?	1
Exercises	4
1.1 Quantum dice	4
1.2 Probability distributions	5
1.3 Waiting times	6
1.4 Stirling's approximation	7
1.5 Stirling and asymptotic series	7
1.6 Random matrix theory	8
1.7 Six degrees of separation	9
1.8 Satisfactory map colorings	12
2 Random walks and emergent properties	15
2.1 Random walk examples: universality and scale invariance	15
2.2 The diffusion equation	19
2.3 Currents and external forces	20
2.4 Solving the diffusion equation	22
2.4.1 Fourier	23
2.4.2 Green	23
Exercises	25
2.1 Random walks in grade space	25
2.2 Photon diffusion in the Sun	26
2.3 Molecular motors and random walks	26
2.4 Perfume walk	27
2.5 Generating random walks	28
2.6 Fourier and Green	28
2.7 Periodic diffusion	29
2.8 Thermal diffusion	30
2.9 Frying pan	30
2.10 Polymers and random walks	30
2.11 Stocks, volatility, and diversification	31
2.12 Computational finance: pricing derivatives	32
2.13 Building a percolation network	33
3 Temperature and equilibrium	37
3.1 The microcanonical ensemble	37
3.2 The microcanonical ideal gas	39
3.2.1 Configuration space	39

3.2.2 Momentum space	41	5.9 Chaos, Lyapunov, and entropy increase	96
3.3 What is temperature?	44	5.10 Entropy increases: diffusion	97
3.4 Pressure and chemical potential	47	5.11 Entropy of glasses	97
3.4.1 Advanced topic: pressure in mechanics and statistical mechanics.	48	5.12 Rubber band	98
3.5 Entropy, the ideal gas, and phase-space refinements	51	5.13 How many shuffles?	99
Exercises	53	5.14 Information entropy	100
3.1 Temperature and energy	54	5.15 Shannon entropy	100
3.2 Large and very large numbers	54	5.16 Fractal dimensions	101
3.3 Escape velocity	54	5.17 Deriving entropy	102
3.4 Pressure computation	54		
3.5 Hard sphere gas	55		
3.6 Connecting two macroscopic systems	55		
3.7 Gas mixture	56		
3.8 Microcanonical energy fluctuations	56		
3.9 Gauss and Poisson	57		
3.10 Triple product relation	58		
3.11 Maxwell relations	58		
3.12 Solving differential equations: the pendulum	58		
4 Phase-space dynamics and ergodicity	63	6 Free energies	105
4.1 Liouville's theorem	63	6.1 The canonical ensemble	106
4.2 Ergodicity	65	6.2 Uncoupled systems and canonical ensembles	109
Exercises	69	6.3 Grand canonical ensemble	112
4.1 Equilibration	69	6.4 What is thermodynamics?	113
4.2 Liouville vs. the damped pendulum	70	6.5 Mechanics: friction and fluctuations	117
4.3 Invariant measures	70	6.6 Chemical equilibrium and reaction rates	118
4.4 Jupiter! and the KAM theorem	72	6.7 Free energy density for the ideal gas	121
5 Entropy	77	Exercises	123
5.1 Entropy as irreversibility: engines and the heat death of the Universe	77	6.1 Exponential atmosphere	124
5.2 Entropy as disorder	81	6.2 Two-state system	125
5.2.1 Entropy of mixing: Maxwell's demon and osmotic pressure	82	6.3 Negative temperature	125
5.2.2 Residual entropy of glasses: the roads not taken	83	6.4 Molecular motors and free energies	126
5.3 Entropy as ignorance: information and memory	85	6.5 Laplace	127
5.3.1 Non-equilibrium entropy	86	6.6 Lagrange	128
5.3.2 Information entropy	87	6.7 Legendre	128
Exercises	90	6.8 Euler	128
5.1 Life and the heat death of the Universe	91	6.9 Gibbs–Duhem	129
5.2 Burning information and Maxwellian demons	91	6.10 Clausius–Clapeyron	129
5.3 Reversible computation	93	6.11 Barrier crossing	129
5.4 Black hole thermodynamics	93	6.12 Michaelis–Menten and Hill	131
5.5 Pressure–volume diagram	94	6.13 Pollen and hard squares	132
5.6 Carnot refrigerator	95	6.14 Statistical mechanics and statistics	133
5.7 Does entropy increase?	95		
5.8 The Arnol'd cat map	95		
		7 Quantum statistical mechanics	135
		7.1 Mixed states and density matrices	135
		7.1.1 Advanced topic: density matrices.	136
		7.2 Quantum harmonic oscillator	139
		7.3 Bose and Fermi statistics	140
		7.4 Non-interacting bosons and fermions	141
		7.5 Maxwell–Boltzmann ‘quantum’ statistics	144
		7.6 Black-body radiation and Bose condensation	146
		7.6.1 Free particles in a box	146
		7.6.2 Black-body radiation	147
		7.6.3 Bose condensation	148
		7.7 Metals and the Fermi gas	150
		Exercises	151
		7.1 Ensembles and quantum statistics	151

7.2 Phonons and photons are bosons	152	9.2 Topological defects in the XY model	204
7.3 Phase-space units and the zero of entropy	153	9.3 Defect energetics and total divergence terms	205
7.4 Does entropy increase in quantum systems?	153	9.4 Domain walls in magnets	206
7.5 Photon density matrices	154	9.5 Landau theory for the Ising model	206
7.6 Spin density matrix	154	9.6 Symmetries and wave equations	209
7.7 Light emission and absorption	154	9.7 Superfluid order and vortices	210
7.8 Einstein's A and B	155	9.8 Superfluids: density matrices and ODLRO	211
7.9 Bosons are gregarious: superfluids and lasers	156		
7.10 Crystal defects	157		
7.11 Phonons on a string	157		
7.12 Semiconductors	157		
7.13 Bose condensation in a band	158		
7.14 Bose condensation: the experiment	158		
7.15 The photon-dominated Universe	159		
7.16 White dwarfs, neutron stars, and black holes	161		
8 Calculation and computation	163	10 Correlations, response, and dissipation	215
8.1 The Ising model	163	10.1 Correlation functions: motivation	215
8.1.1 Magnetism	164	10.2 Experimental probes of correlations	217
8.1.2 Binary alloys	165	10.3 Equal-time correlations in the ideal gas	218
8.1.3 Liquids, gases, and the critical point	166	10.4 Onsager's regression hypothesis and time correlations	220
8.1.4 How to solve the Ising model	166	10.5 Susceptibility and linear response	222
8.2 Markov chains	167	10.6 Dissipation and the imaginary part	223
8.3 What is a phase? Perturbation theory	171	10.7 Static susceptibility	224
Exercises	174	10.8 The fluctuation-dissipation theorem	227
8.1 The Ising model	174	10.9 Causality and Kramers-Krönig	229
8.2 Ising fluctuations and susceptibilities	174	Exercises	231
8.3 Waiting for Godot, and Markov	175	10.1 Microwave background radiation	231
8.4 Red and green bacteria	175	10.2 Pair distributions and molecular dynamics	233
8.5 Detailed balance	176	10.3 Damped oscillator	235
8.6 Metropolis	176	10.4 Spin	236
8.7 Implementing Ising	176	10.5 Telegraph noise in nanojunctions	236
8.8 Wolff	177	10.6 Fluctuation-dissipation: Ising	237
8.9 Implementing Wolff	177	10.7 Noise and Langevin equations	238
8.10 Stochastic cells	178	10.8 Magnetic dynamics	238
8.11 The repressilator	179	10.9 Quasiparticle poles and Goldstone's theorem	239
8.12 Entropy increases! Markov chains	182		
8.13 Hysteresis and avalanches	182		
8.14 Hysteresis algorithms	185		
8.15 NP-completeness and kSAT	186		
9 Order parameters, broken symmetry, and topology	191	11 Abrupt phase transitions	241
9.1 Identify the broken symmetry	192	11.1 Stable and metastable phases	241
9.2 Define the order parameter	192	11.2 Maxwell construction	243
9.3 Examine the elementary excitations	196	11.3 Nucleation: critical droplet theory	244
9.4 Classify the topological defects	198	11.4 Morphology of abrupt transitions	246
Exercises	203	11.4.1 Coarsening	246
9.1 Topological defects in nematic liquid crystals	203	11.4.2 Martensites	250
		11.4.3 Dendritic growth	250
		Exercises	251
		11.1 Maxwell and van der Waals	251
		11.2 The van der Waals critical point	252
		11.3 Interfaces and van der Waals	252
		11.4 Nucleation in the Ising model	253
		11.5 Nucleation of dislocation pairs	254
		11.6 Coarsening in the Ising model	255
		11.7 Origami microstructure	255
		11.8 Minimizing sequences and microstructure	258
		11.9 Snowflakes and linear stability	259

12 Continuous phase transitions	263
12.1 Universality	265
12.2 Scale invariance	272
12.3 Examples of critical points	277
12.3.1 Equilibrium criticality: energy versus entropy	278
12.3.2 Quantum criticality: zero-point fluctuations versus energy	278
12.3.3 Dynamical systems and the onset of chaos	279
12.3.4 Glassy systems: random but frozen	280
12.3.5 Perspectives	281
Exercises	282
12.1 Ising self-similarity	282
12.2 Scaling and corrections to scaling	282
12.3 Scaling and coarsening	282
12.4 Bifurcation theory	283
12.5 Mean-field theory	284
12.6 The onset of lasing	284
12.7 Renormalization-group trajectories	285
12.8 Superconductivity and the renormalization group	286
12.9 Period doubling	288
12.10 The renormalization group and the central limit theorem: short	291
12.11 The renormalization group and the central limit theorem: long	291
12.12 Percolation and universality	293
12.13 Hysteresis and avalanches: scaling	296
A Appendix: Fourier methods	299
A.1 Fourier conventions	299
A.2 Derivatives, convolutions, and correlations	302
A.3 Fourier methods and function space	303
A.4 Fourier and translational symmetry	305
Exercises	307
A.1 Sound wave	307
A.2 Fourier cosines	307
A.3 Double sinusoid	307
A.4 Fourier Gaussians	308
A.5 Uncertainty	309
A.6 Fourier relationships	309
A.7 Aliasing and windowing	310
A.8 White noise	311
A.9 Fourier matching	311
A.10 Gibbs phenomenon	311
References	313
Index	323