

# CONTENTS

## CHAPTER I

### INFINITE SERIES, PRODUCTS, AND INTEGRALS

1.1. Uniform convergence of series . . . . .	2
1.2. Series of complex terms. Power series . . . . .	8
1.3. Series which are not uniformly convergent . . . . .	11
1.4. Infinite products . . . . .	13
1.5. Infinite integrals . . . . .	19
1.6. Double series . . . . .	27
1.7. Integration of series . . . . .	36
1.8. Repeated integrals. The Gamma-function . . . . .	48
1.88. Differentiation of integrals . . . . .	59

## CHAPTER II

### ANALYTIC FUNCTIONS

2.1. Functions of a complex variable . . . . .	64
2.2. The complex differential calculus . . . . .	70
2.3. Complex integration. Cauchy's theorem . . . . .	71
2.4. Cauchy's integral. Taylor's series . . . . .	80
2.5. Cauchy's inequality. Liouville's theorem . . . . .	84
2.6. The zeros of an analytic function . . . . .	87
2.7. Laurent series. Singularities . . . . .	89
2.8. Series and integrals of analytic functions . . . . .	95
2.9. Remark on Laurent Series . . . . .	101

## CHAPTER III

### RESIDUES, CONTOUR INTEGRATION, ZEROS

3.1. Residues. Contour integration . . . . .	102
3.2. Meromorphic functions. Integral functions . . . . .	110
3.3. Summation of certain series . . . . .	114
3.4. Poles and zeros of a meromorphic function . . . . .	115
3.5. The modulus, and real and imaginary parts, of an analytic function . . . . .	119
3.6. Poisson's integral. Jensen's theorem . . . . .	124
3.7. Carleman's theorem . . . . .	130
3.8. A theorem of Littlewood . . . . .	132

## CHAPTER IV

### ANALYTIC CONTINUATION

4.1. General theory . . . . .	138
4.2. Singularities . . . . .	143
4.3. Riemann surfaces. . . . .	146

4.4. Functions defined by integrals. The Gamma-function. The Zeta-function . . . . .	147
4.5. The principle of reflection . . . . .	155
4.6. Hadamard's multiplication theorem . . . . .	157
4.7. Functions with natural boundaries . . . . .	159

## CHAPTER V

## THE MAXIMUM-MODULUS THEOREM

5.1. The maximum-modulus theorem . . . . .	165
5.2. Schwarz's theorem. Vitali's theorem. Montel's theorem . . . . .	168
5.3. Hadamard's three-circles theorem . . . . .	172
5.4. Mean values of $ f(z) $ . . . . .	173
5.5. The Borel-Carathéodory inequality . . . . .	174
5.6. The Phragmén-Lindelöf theorems . . . . .	176
5.7. The Phragmén-Lindelöf function $h(\theta)$ . . . . .	181
5.8. Applications . . . . .	185

## CHAPTER VI

## CONFORMAL REPRESENTATION

6.1. General theory . . . . .	188
6.2. Linear transformations . . . . .	190
6.3. Various transformations . . . . .	195
6.4. Simple ( <i>schlicht</i> ) functions . . . . .	198
6.5. Application of the principle of reflection . . . . .	203
6.6. Representation of a polygon on a half-plane . . . . .	205
6.7. General existence theorems . . . . .	207
6.8. Further properties of simple functions . . . . .	209

## CHAPTER VII

POWER SERIES WITH A FINITE RADIUS  
OF CONVERGENCE

7.1. The circle of convergence . . . . .	213
7.2. Position of the singularities . . . . .	214
7.3. Convergence of the series and regularity of the function . . . . .	217
7.4. Over-convergence. Gap theorems . . . . .	220
7.5. Asymptotic behaviour near the circle of convergence . . . . .	224
7.6. Abel's theorem and its converse . . . . .	229
7.7. Partial sums of a power series . . . . .	235
7.8. The zeros of partial sums . . . . .	238

## CHAPTER VIII

## INTEGRAL FUNCTIONS

8.1. Factorization of integral functions . . . . .	246
8.2. Functions of finite order . . . . .	248

## CONTENTS

ix

8.3. The coefficients in the power series . . . . .	253
8.4. Examples . . . . .	255
8.5. The derived function . . . . .	265
8.6. Functions with real zeros only . . . . .	268
8.7. The minimum modulus . . . . .	273
8.8. The $\alpha$ -points of an integral function. Picard's theorem . . . . .	277
8.9. Meromorphic functions . . . . .	284 <sup>b</sup>

## CHAPTER IX

## DIRICHLET SERIES

9.1. Introduction. Convergence. Absolute convergence . . . . .	289
9.2. Convergence of the series and regularity of the function . . . . .	294
9.3. Asymptotic behaviour . . . . .	295
9.4. Functions of finite order . . . . .	298
9.5. The mean-value formula and half-plane . . . . .	303
9.6. The uniqueness theorem. Zeros . . . . .	309
9.7. Representation of functions by Dirichlet series . . . . .	313

## CHAPTER X

## THE THEORY OF MEASURE AND THE LEBESGUE INTEGRAL

10.1. Riemann integration . . . . .	318
10.2. Sets of points. Measure . . . . .	319
10.3. Measurable functions . . . . .	330
10.4. The Lebesgue integral of a bounded function . . . . .	332
10.5. Bounded convergence . . . . .	337
10.6. Comparison between Riemann and Lebesgue integrals . . . . .	339
10.7. The Lebesgue integral of an unbounded function . . . . .	341
10.8. General convergence theorems . . . . .	345
10.9. Integrals over an infinite range . . . . .	347

## CHAPTER XI

## DIFFERENTIATION AND INTEGRATION

11.1. Introduction . . . . .	349
11.2. Differentiation throughout an interval. Non-differentiable functions . . . . .	350
11.3. The four derivates of a function . . . . .	354
11.4. Functions of bounded variation . . . . .	355
11.5. Integrals . . . . .	359
11.6. The Lebesgue set . . . . .	362
11.7. Absolutely continuous functions . . . . .	364
11.8. Integration of a differential coefficient . . . . .	367

## CONTENTS

## CHAPTER XII

## FURTHER THEOREMS ON LEBESGUE INTEGRATION

12.1. Integration by parts . . . . .	375
12.2. Approximation to an integrable function. Change of the independent variable . . . . .	376
12.3. The second mean-value theorem . . . . .	379
12.4. The Lebesgue class $L^p$ . . . . .	381
12.5. Mean convergence . . . . .	386
12.6. Repeated integrals . . . . .	390

## CHAPTER XIII

## FOURIER SERIES

13.1. Trigonometrical series and Fourier series . . . . .	399
13.2. Dirichlet's integral. Convergence tests . . . . .	402
13.3. Summation by arithmetic means . . . . .	411
13.4. Continuous functions with divergent Fourier series . . . . .	416
13.5. Integration of Fourier series. Parseval's theorem . . . . .	419
13.6. Functions of the class $L^2$ . Bessel's inequality. The Riesz-Fischer theorem . . . . .	422
13.7. Properties of Fourier coefficients . . . . .	425
13.8. Uniqueness of trigonometrical series . . . . .	427
13.9. Fourier integrals . . . . .	432
<b>BIBLIOGRAPHY</b> . . . . .	445
<b>GENERAL INDEX</b> . . . . .	453