CONTENTS

1	**The Standard Model and Beyond**	1
	1.1 The Expanding Universe	1
	1.2 The Hubble Diagram	7
	1.3 Big Bang Nucleosynthesis	9
	1.4 The Cosmic Microwave Background	13
	1.5 Beyond the Standard Model	14
	1.6 Summary	19
	Exercises	21
2	**The Smooth, Expanding Universe**	23
	2.1 General Relativity	23
	2.1.1 The Metric	24
	2.1.2 The Geodesic Equation	28
	2.1.3 Einstein Equations	32
	2.2 Distances	33
	2.3 Evolution of Energy	37
	2.4 Cosmic Inventory	40
	2.4.1 Photons	40
	2.4.2 Baryons	41
	2.4.3 Matter	42
	2.4.4 Neutrinos	44
	2.4.5 Dark Energy	47
	2.4.6 Epoch of Matter–Radiation Equality	50
	2.5 Summary	51
	Exercises	53
3	**Beyond Equilibrium**	58
	3.1 Boltzmann Equation for Annihilation	59
	3.2 Big Bang Nucleosynthesis	62
	3.2.1 Neutron Abundance	65
	3.2.2 Light Element Abundances	68
	3.3 Recombination	70
	3.4 Dark Matter	73
	3.5 Summary	78
	Exercises	80

4 The Boltzmann Equations 84
4.1 The Boltzmann Equation for the Harmonic Oscillator 85
4.2 The Collisionless Boltzmann Equation for Photons 87
4.2.1 Zero-Order Equation 93
4.2.2 First-Order Equation 94
4.3 Collision Terms: Compton Scattering 95
4.4 The Boltzmann Equation for Photons 100
4.5 The Boltzmann Equation for Cold Dark Matter 102
4.6 The Boltzmann Equation for Baryons 106
4.7 Summary 110
Exercises 113

5 Einstein Equations 117
5.1 The Perturbed Ricci Tensor and Scalar 117
5.1.1 Christoffel Symbols 118
5.1.2 Ricci Tensor 119
5.2 Two Components of the Einstein Equations 121
5.3 Tensor Perturbations 124
5.3.1 Christoffel Symbols for Tensor Perturbations 125
5.3.2 Ricci Tensor for Tensor Perturbations 127
5.3.3 Einstein Equations for Tensor Perturbations 129
5.4 The Decomposition Theorem 131
5.5 From Gauge to Gauge 132
5.6 Summary 135
Exercises 136

6 Initial Conditions 139
6.1 The Einstein–Boltzmann Equations at Early Times 139
6.2 The Horizon 142
6.3 Inflation 144
6.3.1 A Solution to the Horizon Problem 146
6.3.2 Negative Pressure 151
6.3.3 Implementation with a Scalar Field 151
6.4 Gravity Wave Production 155
6.4.1 Quantizing the Harmonic Oscillator 156
6.4.2 Tensor Perturbations 157
6.5 Scalar Perturbations 162
6.5.1 Scalar Field Perturbations around a Smooth Background 162
6.5.2 Super-Horizon Perturbations 164
6.5.3 Spatially Flat Slicing 169
6.6 Summary and Spectral Indices 170
Exercises 175

7 Inhomogeneities 180
7.1 Prelude 180

7.1.1 Three Stages of Evolution 182
7.1.2 Method 185
7.2 Large Scales 189
7.2.1 Super-horizon Solution 189
7.2.2 Through Horizon Crossing 192
7.3 Small Scales 194
7.3.1 Horizon Crossing 195
7.3.2 Sub-horizon Evolution 199
7.4 Numerical Results and Fits 203
7.5 Growth Function 205
7.6 Beyond Cold Dark Matter 207
7.6.1 Baryons 208
7.6.2 Massive Neutrinos 209
7.6.3 Dark Energy 210
Exercises 212

Anisotropies 216
8.1 Overview 217
8.2 Large-Scale Anisotropies 223
8.3 Acoustic Oscillations 224
8.3.1 Tightly Coupled Limit of the Boltzmann Equations 224
8.3.2 Tightly Coupled Solutions 227
8.4 Diffusion Damping 230
8.5 Inhomogeneities to Anisotropies 234
8.5.1 Free Streaming 234
8.5.2 The C_l's 239
8.6 The Anisotropy Spectrum Today 242
8.6.1 Sachs–Wolfe Effect 242
8.6.2 Small Scales 245
8.7 Cosmological Parameters 248
8.7.1 Curvature 249
8.7.2 Degenerate Parameters 251
8.7.3 Distinct Imprints 253
Exercises 256

Probes of Inhomogeneities 261
9.1 Angular Correlations 262
9.2 Peculiar Velocities 270
9.3 Direct Measurements of Peculiar Velocities 271
9.4 Redshift Space Distortions 275
9.5 Galaxy Clusters 282
Exercises 289

Weak Lensing and Polarization 292
10.1 Gravitational Distortion of Images 293

10.2 Geodesics and Shear 296
10.3 Ellipticity as an Estimator of Shear 300
10.4 Weak Lensing Power Spectrum 302
10.5 Polarization: The Quadrupole and the Q/U Decomposition 310
10.6 Polarization from a Single Plane Wave 313
10.7 Boltzmann Solution 320
10.8 Polarization Power Spectra 323
10.9 Detecting Gravity Waves 326
Exercises 331

11 Analysis 336
11.1 The Likelihood Function 337
11.1.1 Simple Example 337
11.1.2 CMB Likelihood 340
11.1.3 Galaxy Surveys 343
11.2 Signal Covariance Matrix 344
11.2.1 CMB Window Functions 345
11.2.2 Examples of CMB Window Functions 347
11.2.3 Window Functions for Galaxy Surveys 350
11.2.4 Summary 354
11.3 Estimating the Likelihood Function 356
11.3.1 Karhunen–Loève Techniques 356
11.3.2 Optimal Quadratic Estimator 362
11.4 The Fisher Matrix: Limits and Applications 368
11.4.1 CMB 368
11.4.2 Galaxy Surveys 370
11.4.3 Forecasting 371
11.5 Mapmaking and Inversion 375
11.6 Systematics 378
11.6.1 Foregrounds 378
11.6.2 Mode Subtraction 384
Exercises 389

A Solutions to Selected Problems 392

B Numbers 415
B.1 Physical Constants 415
B.2 Cosmological Constants 416

C Special Functions 418
C.1 Legendre Polynomials 418
C.2 Spherical Harmonics 418
C.3 Spherical Bessel Functions 419
C.4 Fourier Transforms 420
C.5 Miscellaneous 420

D Symbols 422

Bibliography 426

Index 435