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Preface

This book has evolved from lecture notes for a course in general relativity which was my own experience which firmly convinced me that it was significantly more difficult for undergraduates than the undergraduate-level treatments of special relativity and mechanics. The explosion of research in general relativity in the past 20 years, largely stimulated by the discovery of black holes, has deepened and made more complete our understanding of the subject in much simpler, more physical ways than were possible in the mainstream of physics and astronomy until recently. It can be regarded as broadly educational for physicists. The formidable reputation relative to other areas of physics in the viewer: 'Professor Eddington, in the world understand Einstein's theory of relativity today perhaps the chief obstacle to the acceptance of relativity to theoretical physicists. The aim of this book is to present relativity at a level appropriate for students who will understand the basic physical concepts and, with appropriate qualifications, will be able to solve elementary problems. Those prepared for the more advanced topics will find this book a useful reference.

In pursuing this aim, I have tried to make the book as self-contained as possible. First, to assume a minimum of pre-