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FOREWORD TO F

When my young colleagues, Jean
invited me to write a few introd
publication of their work *Mécaniqu*
solids materials), I very willingly acc
trust and friendship, even though o
is made abundantly clear by the title
the well-deserved fame of the autho
the attention of readers.

The originality, I would even s
apparent only if we place it within
during the last few decades. In fact, i
developments it assimilates and in
continuum mechanics and thermod
methods which, starting with a few s
great variety of phenomenological
extremely varied behaviour of solid
above all, takes advantage of the ob
and empirical laws which genera
imagination and perspicacity in gui
Finally, it provides an inventory of
observed at the microscopic, molecu
events determining and explaining
these phenomena cannot be exp
formulas, they are mentioned, whe
results and procedures.

To my mind, this triple heritage ca
development of continuum thermod