

# Contents

*Preface*    *xi*

**1 Preliminaries**    **1**

- 1-1. Vector Spaces    1
- 1-2. Linear Transformations and Eigenvectors    4
- 1-3. Orientation and Cross Products    6
- 1-4. Lines, Planes, and Spheres    8
- 1-5. Vector Calculus    10

**2 Local Curve Theory**    **13**

- 2-1. Basic Definitions and Examples    14
- 2-2. Arc Length    20
- 2-3. Curvature and the Frenet-Serret Apparatus    24
- 2-4. The Frenet-Serret Theorem and Its Corollaries    29
- 2-5. The Fundamental Existence and Uniqueness Theorem for Curves    41
- 2-6. Non-Unit Speed Curves    46

<b>3</b>	<b><i>Global Theory of Plane Curves</i></b>	<b>49</b>	<b>7</b>	<b>IA</b>
	3-1. Line Integrals and Green's Theorem	50		7
	3-2. The Rotation Index of a Plane Curve	52		7
	3-3. Convex Curves	60		7
	3-4. The Isoperimetric Inequality	63		71
	3-5. The Four-Vertex Theorem	66		71
	3-6. A Preview	72		71
<b>4</b>	<b><i>Local Surface Theory</i></b>	<b>74</b>		74
	4-1. Basic Definitions and Examples	76		
	4-2. Surfaces	88		
	4-3. The First Fundamental Form and Arc Length	93		<b>A</b>
	4-4. Normal Curvature, Geodesic Curvature, and Gauss's Formulas	102		
	4-5. Geodesics	709		<b>B</b>
	4-6. Parallel Vector Fields Along a Curve and Parallelism	716		
	4-7. The Second Fundamental Form and the Weingarten Map	122		<b>In</b>
	4-8. Principal, Gaussian, Mean, and Normal Curvatures	127		IJ
	4-g. Riemannian Curvature and Gauss's <i>Theorema Egregium</i>	141		II.
	4-10. Isometries and the Fundamental Theorem of Surfaces	746		
	4-1 1. Surfaces of Constant Curvature	153		
<b>5</b>	<b><i>Global Theory of Space Curves</i></b>	<b>767</b>		
	5-1. Fenchel's Theorem	161		
	5-2. The Fary-Milnor Theorem	167		
	5-3. Total Torsion	170		
<b>6</b>	<b><i>Global Theory of Surfaces</i></b>	<b>773</b>		
	6-1. Simple Curvature Results	174		
	6-2. Geodesic Coordinate Patches	176		
	6-3. Orientability and Angular Variation	180		
	6-4. The Gauss-Bonnet Formula	785		
	6-5. The Gauss-Bonnet Theorem and the Euler Characteristic	788		
	6-6. The Theorems of Jacobi and Hadamard	792		
	6-7. The Index of a Vector Field	795		

<b>7</b>	<b><i>Introduction to Manifolds</i></b>	<b>198</b>
7-1.	Some Analytic Preliminaries	199
7-2.	Manifolds—Definition and Examples	203
7-3.	Tangent Vectors and the Tangent Space	208
7-4.	Vector Fields and Lie Brackets	216
7-5.	The Differential of a Map and Submanifolds	219
7-6.	Linear Connections on Manifolds	223
7-7.	Parallel Vector Fields and Geodesics on a Manifold with a Linear Connection	227
7-8.	Riemannian Metrics, Distance, and Curvature	232

	<b><i>Appendix: Historical Notes</i></b>	<b>243</b>
--	--	------------

	<b><i>Bibliography</i></b>	<b>248</b>
--	----------------------------	------------

### ***Index***

I.	Notational	255
II.	Topical	257

\_\_\_\_\_

|