

Contents

Preface	xi
List of Boxes	xv
1 Introduction	1
1.1 Nonlinear finite elements in design	1
1.2 Related books and a brief history of nonlinear finite elements	4
1.3 Notation	7
1.4 Mesh descriptions	9
1.5 Classification of partial differential equations	13
1.6 Exercises	18
2 Lagrangian and Eulerian finite elements in one dimension	19
2.1 Introduction	19
2.2 Governing equations for total Lagrangian formulation	20
2.3 Weak form for total Lagrangian formulation	27
2.4 Finite element discretization in total Lagrangian formulation	33
2.5 Element and global matrices	38
2.6 Governing equations for updated Lagrangian formulation	48
2.7 Weak form for updated Lagrangian formulation	51
2.8 Element equations for updated Lagrangian formulation	52
2.9 Governing equations for Eulerian formulation	64
2.10 Weak forms for Eulerian mesh equations	65
2.11 Finite element equations	66
2.12 Solution methods	70
2.13 Summary	72
2.14 Exercises	72
3 Continuum mechanics	75
3.1 Introduction	75
3.2 Deformation and motion	76
3.3 Strain measures	92
3.4 Stress measures	101
3.5 Conservation equations	108
3.6 Lagrangian conservation equations	119
3.7 Polar decomposition and frame-invariance	125
3.8 Exercises	137

4 Lagrangian meshes	141	8 Elen
4.1 Introduction	141	8.1
4.2 Governing equations	142	8.2
4.3 Weak form: principle of virtual power	145	8.3
4.4 Updated Lagrangian finite element discretization	152	8.4
4.5 Implementation	162	8.5
4.6 Corotational formulations	185	8.6
4.7 Total Lagrangian formulation	i93	8.7
4.8 Total Lagrangian weak form	196	8.8
4.9 Finite element semidiscretization	198	8.9
4.10 Exercise	213	8.10
5 Constitutive models	215	9 Beam
5.1 Introduction	215	9.1 ,
5.2 The stress-strain curve	216	9.2 1
5.3 One-dimensional elasticity	221	9.3 C
5.4 Nonlinear elasticity	225	9.4 A
5.5 One-dimensional plasticity	240	9.5 C
5.6 Multiaxial plasticity	247	9.6 D
5.7 Hyperelastic-plastic models	264	9.7 S
5.8 Viscoelasticity	274	9.8 P
5.9 Stress update algorithms	277	9.9 D
5.10 Continuum mechanics and constitutive models	294	9.10 E
5.11 Exercises	308	
6 Solution methods and stability	309	10 Contact
6.1 Introduction	309	10.1 Ir
6.2 Explicit methods	310	10.2 c
6.3 Equilibrium solutions and implicit time integration	317	10.3 F
6.4 Linearization	337	10.4 W
6.5 Stability and continuation methods	353	10.5 Fi
6.6 Numerical stability	369	10.6 O
6.7 Material stability	384	
6.8 Exercises	392	
7 Arbitrary Lagrangian Eulerian formulations	393	Appendix 1
7.1 Introduction	393	Appendix 2
7.2 ALE continuum mechanics	395	Appendix 3
7.3 Conservation laws in ALE description	402	Glossary
7.4 ALE governing equations	403	References
7.5 Weak forms	404	
7.6 Introduction to the Petrov-Galerkin method	408	
7.7 Petrov-Galerkin formulation of momentum equation	417	
7.8 Path-dependent materials	420	
7.9 Linearization of the discrete equations	432	
7.10 Mesh update equations	435	
7.11 Numerical example: an elastic-plastic wave propagation problem	442	
7.12 Total ALE formulations	443	

8 Element technology	451
8.1 Introduction	451
8.2 Element performance	453
8.3 Element properties and patch tests	461
8.4 Q4 and volumetric locking	469
8.5 Multi-field weak forms and elements	474
8.6 Multi-field quadrilaterals	487
8.7 One-point quadrature elements	491
8.8 Examples	500
8.9 Stability	504
8.10 Exercises	507
9 Beams and shells	509
9.1 Introduction	509
9.2 Beam theories	511
9.3 Continuum-based beam	514
9.4 Analysis of CB beam	524
9.5 Continuum-based shell implementation	536
9.6 CB shell theory	550
9.7 Shear and membrane locking	555
9.8 Assumed strain elements	560
9.9 One-point quadrature elements	563
9.10 Exercises	566
10 Contact-impact	569
10.1 Introduction	569
10.2 Contact interface equations	570
10.3 Friction models	580
10.4 Weak forms	585
10.5 Finite element discretization	595
10.6 On explicit methods	609
Appendix 1 Voigt notation	615
Appendix 2 Norms	619
Appendix 3 Element shape functions	622
Glossary	627
References	631
Index	641