
Contents

Preface	<i>page xvii</i>
Part A: The Fundamentals of MHD	1
Introduction: The Aims of Part A	1
1 A Qualitative Overview of MHD	3
1.1 What is MHD?	3
1.2 A Brief History of MHD	6
1.3 From Electrodynamics to MHD: A Simple Experiment	8
1.3.1 Some important parameters in electrodynamics and MHD	8
1.3.2 A brief reminder of the laws of electrodynamics	9
1.3.3 A familiar high-school experiment	11
1.3.4 A summary of the key results for MHD	18
1.4 Some Simple Applications of MHD	18
2 The Governing Equations of Electrodynamics	27
2.1 The Electric Field and the Lorentz Force	27
2.2 Ohm's Law and the Volumetric Lorentz Force	29
2.3 Ampere's Law	31
2.4 Faraday's Law in Differential Form	32
2.5 The Reduced Form of Maxwell's Equations for MHD	34
2.6 A Transport Equation for \mathbf{B}	37
2.7 On the Remarkable Nature of Faraday and of Faraday's Law	37
2.7.1 An historical footnote	37
2.7.2 An important kinematic equation	40

2.7.3	The full significance of Faraday's law	42
2.7.4	Faraday's law in ideal conductors: Alfvén's theorem	44
3	The Governing Equations of Fluid Mechanics	47
	Part 1: Fluid Flow in the Absence of Lorentz Forces	47
3.1	Elementary Concepts	47
3.1.1	Different categories of fluid flow	47
3.1.2	The Navier-Stokes equation	59
3.2	Vorticity, Angular Momentum and the Biot-Savart Law	61
3.3	Advection and Diffusion of Vorticity	64
3.3.1	The vorticity equation	64
3.3.2	Advection and diffusion of vorticity: temperature as a prototype	66
3.3.3	Vortex line stretching	70
3.4	Kelvin's Theorem, Helmholtz's Laws and Helicity	71
3.4.1	Kelvin's Theorem and Helmholtz's Laws	71
3.4.2	Helicity	74
3.5	The Prandtl-Batchelor Theorem	77
3.6	Boundary Layers, Reynolds Stresses and Turbulence Models	81
3.6.1	Boundary layers	81
3.6.2	Reynolds stresses and turbulence models	83
3.7	Ekman Pumping in Rotating Flows	90
	Part 2: Incorporating the Lorentz Force	95
3.8	The Full Equations of MHD and Key Dimensionless Groups	95
3.9	Maxwell Stresses	97
4	Kinematics of MHD: Advection and Diffusion of a Magnetic Field	102
4.1	The Analogy to Vorticity	102
4.2	Diffusion of a Magnetic Field	103
4.3	Advection in Ideal Conductors: Alfvén's Theorem	104
4.3.1	Alfvén's theorem	104
4.3.2	An aside: sunspots	106
4.4	Magnetic Helicity	108
4.5	Advection plus Diffusion	109
4.5.1	Field sweeping	109
4.5.2	Flux expulsion	110

4.5.3	Azimuthal field generation by differential rotation	114
4.5.4	Magnetic reconnection	115
5	Dynamics at Low Magnetic Reynolds Numbers	117
5.1	The Low- R_m Approximation in MHD	118
	Part 1: Suppression of Motion	119
5.2	Magnetic Damping	119
5.2.1	The destruction of mechanical energy via Joule dissipation	120
5.2.2	The damping of a two-dimensional jet	121
5.2.3	Damping of a vortex	122
5.3	A Glimpse at MHD Turbulence	128
5.4	Natural Convection in the Presence of a Magnetic Field	132
5.4.1	Rayleigh-Bénard convection	132
5.4.2	The governing equations	133
5.4.3	An energy analysis of the Rayleigh-Bénard instability	134
5.4.4	Natural convection in other configurations	137
	Part 2: Generation of Motion	139
5.5	Rotating Fields and Swirling Motions	139
5.5.1	Stirring of a long column of metal	139
5.5.2	Swirling flow induced between two parallel plates	142
5.6	Motion Driven by Current Injection	145
5.6.1	A model problem	145
5.6.2	A useful energy equation	146
5.6.3	Estimates of the induced velocity	148
5.6.4	A paradox	149
	Part 3: Boundary Layers	151
5.7	Hartmann Boundary Layers	151
5.7.1	The Hartmann Layer	151
5.7.2	Hartmann flow between two planes	152
5.8	Examples of Hartmann and Related Flows	154
5.8.1	Flow-meters and MHD generators	154
5.8.2	Pumps, propulsion and projectiles	155
5.9	Conclusion	157

6	Dynamics at Moderate to High Magnetic Reynolds' Number	159
6.1	Alfvén Waves and Magnetostrophic Waves	160
6.1.1	Alfvén waves	160
6.1.2	Magnetostrophic waves	163
6.2	Elements of Geo-Dynamo Theory	166
6.2.1	Why do we need a dynamo theory for the earth?	166
6.2.2	A large magnetic Reynolds number is needed	171
6.2.3	An axisymmetric dynamo is not possible	174
6.2.4	The influence of small-scale turbulence: the α -effect	177
6.2.5	Some elementary dynamical considerations	185
6.2.6	Competing kinematic theories for the geo-dynamo	197
6.3	A Qualitative Discussion of Solar MHD	199
6.3.1	The structure of the sun	200
6.3.2	Is there a solar dynamo?	201
6.3.3	Sunspots and the solar cycle	201
6.3.4	The location of the solar dynamo	203
6.3.5	Solar flares	203
6.4	Energy-Based Stability Theorems for Ideal MHD	206
6.4.1	The need for stability theorems in ideal MHD: plasma containment	207
6.4.2	The energy method for magnetostatic equilibria	208
6.4.3	An alternative method for magnetostatic equilibrium	213
6.4.4	Proof that the energy method provides both necessary and sufficient conditions for stability	215
6.4.5	The stability of non-static equilibria	216
6.5	Conclusion	220
7	MHD Turbulence at Low and High Magnetic Reynolds Number	222
7.1	A Survey of Conventional Turbulence	223
7.1.1	A historical interlude	223
7.1.2	A note on tensor notation	227
7.1.3	The structure of turbulent flows: the Kolmogorov picture of turbulence	229
7.1.4	Velocity correlation functions and the Karman-Howarth equation	235

7.1.5	Decaying turbulence: Kolmogorov's law, Loitsyansky's integral, Landau's angular momentum and Batchelor's pressure forces	240
7.1.6	On the difficulties of direct numerical simulations	247
7.2	MHD Turbulence	249
7.2.1	The growth of anisotropy at low and high R_m	249
7.2.2	Decay laws at low R_m	252
7.2.3	The spontaneous growth of a magnetic field at high R_m	256
7.3	Two-Dimensional Turbulence	260
7.3.1	Batchelor's self-similar spectrum and the inverse energy cascade	260
7.3.2	Coherent vortices	263
7.3.3	The governing equations of two-dimensional turbulence	264
7.3.4	Variational principles for predicting the final state in confined domains	267
	Part B: Applications in Engineering and Metallurgy	273
	Introduction: An Overview of Metallurgical Applications	273
8	Magnetic Stirring Using Rotating Fields	285
8.1	Casting, Stirring and Metallurgy	285
8.2	Early Models of Stirring	289
8.3	The Dominance of Ekman Pumping in the Stirring of Confined Liquids	294
8.4	The Stirring of Steel	298
9	Magnetic Damping Using Static Fields	301
9.1	Metallurgical Applications	301
9.2	Conservation of Momentum, Destruction of Energy and the Growth of Anisotropy	304
9.3	Magnetic Damping of Submerged Jets	308
9.4	Magnetic Damping of Vortices	312
9.4.1	General considerations	312
9.4.2	Damping of transverse vortices	314
9.4.3	Damping of parallel vortices	317
9.4.4	Implications for low- R_m turbulence	323
9.5	Damping of Natural Convection	324

9.5.1	Natural convection in an aluminium ingot	324
9.5.2	Magnetic damping in an aluminium ingot	329
10	Axisymmetric Flows Driven by the Injection of Current	332
10.1	The VAR Process and a Model Problem	332
10.1.1	The VAR process	332
10.1.2	Integral constraints on the flow	336
10.2	The Work Done by the Lorentz Force	338
10.3	Structure and Scaling of the Flow	340
10.3.1	Differences between confined and unconfined flows	340
10.3.2	Shercliff's self-similar solution for unconfined flows	342
10.3.3	Confined flows	3 4 4
10.4	The Influence of Buoyancy	346
10.5	Stability of the Flow and the Apparent Growth of Swirl	348
10.5.1	An extraordinary experiment	348
10.5.2	There is no spontaneous growth of swirl!	350
10.6	Flaws in the Traditional Explanation for the Emergence of Swirl	351
10.7	The Role of Ekman Pumping in Establishing the Dominance of Swirl	353
10.7.1	A glimpse at the mechanisms	353
10.7.2	A formal analysis	356
10.7.3	Some numerical experiments	358
11	MHD Instabilities in Reduction Cells	363
11.1	Interfacial Waves in Aluminium Reduction Cells	363
11.1.1	Early attempts to produce aluminium by electrolysis	363
11.1.2	The instability of modern reduction cells	364
11.2	A Simple Mechanical Analogue for the Instability	368
11.3	Simplifying Assumptions	372
11.4	A Shallow-Water Wave Equation and Key Dimensionless Groups	374
11.4.1	A shallow-water wave equation	374
11.4.2	Key dimensionless groups	378
11.5	Travelling Wave and Standing Wave Instabilities	379
11.5.1	Travelling waves	379
11.5.2	Standing waves in circular domains	380
11.5.3	Standing waves in rectangular domains	381

11.6 Implications for Reduction Cell Design	385
12 High-Frequency Fields: Magnetic Levitation and Induction Heating	387
12.1 The Skin Effect	388
12.2 Magnetic Pressure, Induction Heating and High-Frequency Stirring	390
12.3 Applications in the Casting of Steel, Aluminium and Super-Alloys	394
12.3.1 The induction furnace	394
12.3.2 The cold crucible	397
12.3.3 Levitation melting	398
12.3.4 Processes which rely on magnetic repulsion EM valves and EM casters	403
Appendices	
1 Vector Identities and Theorems	405
2 Stability Criteria for Ideal MHD Based on the Hamiltonian	407
3 Physical Properties of Liquid Metals	417
4 MHD Turbulence at Low R_m	418
Bibliography	422
Suggested Books on Fluid Mechanics	422
Suggested Books on Electromagnetism	422
Suggested Books on MHD	423
Journal References for Part B and Appendix 2	423
Subject Index	427