CONTENTS

Preface

1

2

Suggestions for the Instructor	xi
THE NATURE OF DIFFERENTIAL EQUATIONS	1
1. Introduction	
2. General remarks on solutions	3
3. Families of curves. Orthogonal trajectories	8
4. Growth, decay, and chemical reactions	14
5. Falling bodies and other rate problems	19
6. The brachistochrone. Fermat and the Bernoullis	25
FIRST ORDER EQUATIONS	35
7. Homogeneous equations	35
8. Exact equations	38
9. Integrating factors	42
10. Linear equations	47
11. Reduction of order	49
12. The hanging chain. Pursuit curves	52
13. Simple electric circuits	58
Appendix A. Numerical methods	65

XV

vii

3	SECOND ORDER LINEAR EQUATIONS	72
Ũ	14. Introduction	72
	15. The general solution of the homogeneous equation	76
	16. The use of a known solution to find another	81
	17. The homogeneous equation with constant coefficients	83
	18. The method of undetermined coefficients	87
	19. The method of variation of parameters	90
	20. Vibrations in mechanical systems	93
	21. Newton's law of gravitation and the motion of the planets	100
	Appendix A. Euler	107
	Appendix B. Newton	111
4	OSCILLATION THEORY AND	
	BOUNDARY VALUE PROBLEMS	115
	22. Qualitative properties of solutions	115
	23. The Sturm comparison theorem	121
	24. Eigenvalues, eigenfunctions, and the vibrating string	124
	Appendix A. Regular Sturm-Liouville problems	133
5	POWER SERIES SOLUTIONS AND	
	SPECIAL FUNCTIONS	140
	25. Introduction. A review of power series	140
	26. Series solutions of first order equations	147
	27. Second order linear equations. Ordinary points	151
	28. Regular singular points	159
	29. Regular singular points (continued)	167
	30. Gauss's hypergeometric equation	174
	31 . The point at infinity	1 8
	Appendix A. Two convergence proofs	183
	Appendix B. Hermite polynomials and quantum mechanics	187
	Appendix C. Gauss	196
	Appendix D. Chebyshev polynomials and the minimax property	204
	Appendix E. Riemann's equation	211
6	SOME SPECIAL FUNCTIONS OF	040
	MATHEMATICAL PHYSICS	219
	32. Legendre polynomials	219
	33. Properties of Legendre polynomials	226
	34. Bessel functions. The gamma function	232
	35. Properties of Bessel functions	242
	Appendix A. Legendre polynomials andpotential theory	' 249
	Appendix B. Bessel functions and the vibrating membrane	255
	Appendix C. Additional properties of Bessel functions	261

7	 SYSTEMS OF FIRST ORDER EQUATIONS 36] General remarks on systems 37. Linear systems 38. Homogeneous linear systems with constant coefficients 39] Nonlinear systems. Volerra's prey-predator equations 	265 265 268 276 284
8	 NONLINEAR EQUATIONS 40. Autonomous systems. The phase plane and its phenomena 41. Types of critical points. Stability 42. Critical points and stability for linear systems 43. Stability by Liapunov's direct method 44. Simple critical points of nonlinear systems 46. Nonlinear mechanics. Conservative systems 46. Periodic solutions. The Poincaré-Bendixson theorem Appendix A. Poincaré Appendix B. Proof of Liéard's theorem 	290 296 305 316 323 332 338 346 349
9	 THE CALCULUS OF VARIATIONS 47. Introduction. Some typical problems of the subject 48. Euler's differential equation for an extremal 49. Isoperimetric problems Appendix A. Lagrange Appendix B. Hamilton's principle and its implications 	353 353 356 366 376 377
10	 LAPLACE TRANSFORMS 50. Introduction 51. A few remarks on the theory 52. Applications to differential equations 53. Derivatives and integrals of Laplace transforms 54. Convolutions and Abel's mechanical problem Appendix A. Laplace Appendix B. Abel 	3 8 8 388 392 397 402 407 413 415 415 415
11	 THE EXISTENCE AND UNIQUENESS OF SOLUTIONS 55. The method of successive approximations 56. Picard's theorem 57. Systems. The second order linear equation Answers Index	418 418 422 433 436 457