

# Contents

---

<b>PREFACE FOR THE STUDENT</b>	<b>1</b>
1. A brief lesson in logic . . . . .	1
2. Glossary of technical symbols . . . . .	5
<b>I. THE NUMBER SYSTEM OF ALGEBRA</b>	<b>7</b>
3. Introduction . . . . .	7
4. The natural numbers. Addition and multiplication . . . . .	8
5. Subtraction and division of natural numbers . . . . .	12
6. Even, odd, and prime numbers. H.C.F. and L.C.M. . . . .	15
7. Fractions. Multiplication and division . . . . .	19
8. Fractions. Addition and subtraction . . . . .	23
9. Directed numbers. Addition and multiplication . . . . .	26
10. Directed numbers. Subtraction and division . . . . .	31
11. The system of rational numbers. Properties of zero . . . . .	35
12. Positive integral powers and roots . . . . .	37
13. The square root of 2 . . . . .	41
14. The system of real numbers. Decimal notation . . . . .	44
*15. Extraction of square roots . . . . .	49
16. The system of complex numbers . . . . .	51
*17. Conclusion. The axioms of algebra . . . . .	54
<b>II. WHAT IS ALGEBRA?</b>	<b>57</b>
18. Algebra as a language . . . . .	57
19. Algebra as a logical science . . . . .	58
20. Algebra as a collection of techniques of calculation . . . . .	58
21. Algebra as a branch of human endeavor . . . . .	59
22. Algebra as a collection of puzzles . . . . .	59
<b>III. ALGEBRAIC EXPRESSIONS</b>	<b>61</b>
23. Positive integral exponents and radicals. Order of operations . . . . .	61
24. Algebraic expressions . . . . .	64

\* Sections starred may be omitted without disturbing the continuity of the chapter.

25. Removal and insertion of parentheses. Combination of like terms. . . . .	68
26. Standard form of polynomials . . . . .	69
27. Equations and identities . . . . .	72
28. Verification of identities. Reversible steps. . . . .	74
29. Elementary operations with equations . . . . .	76
30. Standard form of polynomial equations. Degree of an equation . . . . .	78
<b>IV. FUNCTIONS AND GRAPHS</b>	<b>80</b>
31. Functions. . . . .	80
32. Functional notation . . . . .	82
33. Applications of functions in science. Formulas. . . . .	83
34. Rectangular coordinates. . . . .	85
35. The graphs of equations and functions . . . . .	86
36. Continuous functions. . . . .	89
37. Graphical solution of equations . . . . .	90
*38. Infinities of a rational function. . . . .	92
<b>V. ELEMENTARY OPERATIONS WITH POLYNOMIALS</b>	<b>94</b>
39. Integers and polynomials. Hindu-Arabic notation . . . . .	94
40. Addition and subtraction of polynomials . . . . .	95
41. Multiplication of polynomials . . . . .	97
42. Division by monomials . . . . .	99
43. The division algorithm . . . . .	100
*44. Non-decimal scales of notation. . . . .	104
<b>VI. FACTORING OF POLYNOMIALS</b>	<b>110</b>
45. Prime factors . . . . .	110
46. Special products . . . . .	111
47. Simple types of factoring . . . . .	113
48. Factoring by grouping . . . . .	115
49. Quadratic trinomials . . . . .	115
*50. Sum and difference of two like powers . . . . .	117
51. H.C.F. and L.C.M. . . . .	118
<b>VII. ELEMENTARY OPERATIONS WITH FRACTIONAL EXPRESSIONS</b>	<b>120</b>
52. Reduction to lowest terms. . . . .	120
53. Multiplication and division of fractional expressions	122
54. Addition and subtraction of fractional expressions . . . . .	124
55. Complex fractions . . . . .	126

**VIII. LINEAR EQUATIONS AND LINEAR FUNCTIONS** 129

- 56. Solution of linear equations in one variable . . . . . 129
- 57. Equations leading to linear equations. . . . . 131
- 58. Literal equations and formulas. . . . . 133
- 59. Graph of a linear function. . . . . 135
- 60. Systems of two linear equations in two unknowns . 139
- 61. Graphical interpretation. Inconsistent and dependent equations . . . . . 142
- 62. Systems of three linear equations in three unknowns 145
- 63. Systems of equations reducible to linear equations . 147
- 64. Verbal problems . . . . . 150

**IX. INTEGRAL AND FRACTIONAL EXPONENTS** 158

- 65. Negative integers and zero as exponents. . . . . 158
- 66. A use of powers of ten in scientific writing. . . . . 161
- 67. Fractional exponents . . . . . 162

**X. RADICALS** 165

- 68. Basic laws for radicals . . . . . 165
- 69. The simplest form of a radical. . . . . 166
- 70. Addition and subtraction of radicals . . . . . 168
- 71. Multiplication with radicals of the same index. . . 169
- 72. Division with radicals. Rationalizing denominators. 171
- 73. A use of fractional exponents . . . . . 172

**XI. QUADRATIC EQUATIONS AND QUADRATIC FUNCTIONS** 174

- 74. Solution of quadratic equations by factoring. . . . . 174
- 75. Completing the square . . . . . 176
- 76. The quadratic formula . . . . . 178
- 77. Character of the roots . . . . . 180
- 78. The graph of a quadratic function. Graphical solution of quadratic equations . . . . . 182
- 79. The sum and product of the roots . . . . . 184
- 80. Equations leading to quadratics . . . . . 186
- 81. Irrational equations . . . . . 188
- 82. Verbal problems . . . . . 189
- \*83. Maxima and minima of quadratic functions. . . . 192

**XII. SYSTEMS OF EQUATIONS IN TWO UNKNOWN INVOLVING QUADRATICS** 196

- 84. The graph of a quadratic equation in  $x$  and  $y$  . . . 196
- 85. Graphical solution of simultaneous equations . . . 198

86. Algebraic solution when one equation is linear and one quadratic . . . . .	200
87. Algebraic solution when both equations are quadratic . . . . .	202
88. Verbal problems . . . . .	206
<b>XIII] RATIO, PROPORTION, AND VARIATION</b>	<b>208</b>
89. Ratio and proportion. . . . .	208
90. The terminology of variation . . . . .	212
<b>XIV] PERMUTATIONS AND COMBINATIONS</b>	<b>217</b>
91. Fundamental principles. . . . .	217
92. Permutations of $n$ distinct objects taken $r$ at a time.	221
93. Permutations where some things are alike. . . . .	223
94. Circular permutations . . . . .	224
95. Combinations of $n$ distinct objects taken $r$ at a time	225
<b>XV] PROBABILITY</b>	<b>229</b>
96. Definition of probability . . . . .	229
97. Statistical probability. . . . .	235
98. Conditional probabilities. Independent events. Repeated trials. . . . .	236
<b>XVI] MATHEMATICAL INDUCTION</b>	<b>243</b>
99. The $n$ th term of a sequence . . . . .	243
100. The axiom of mathematical induction. . . . .	244
101. Proofs by mathematical induction . . . . .	245
<b>XVII. THE BINOMIAL THEOREM</b>	<b>250</b>
102. The binomial theorem for positive integral exponents	250
*103] Proof of the binomial theorem by mathematical induction. . . . .	255
104. The binomial series. . . . .	256
<b>XVIII] COMPLEX NUMBERS</b>	<b>258</b>
105. The standard form of a complex number . . . . .	258
106. Elementary operations with complex numbers. . . . .	260
107. Graphical representation of complex numbers . . . . .	263
108. Graphical significance of addition of complex numbers . . . . .	265
109. Review of trigonometric functions . . . . .	266

110. Trigonometric or polar form of a complex number .	269
111. Graphical meaning of multiplication and division of complex numbers. . . . .	272
112. De Moivre's theorem. . . . .	274
113. The nth roots of a complex number. . . . .	275
114. Solution of pure equations of nth degree . . . . .	278

**XIX. THEORY OF EQUATIONS** 279

115. The fundamental theorem of algebra . . . . .	279
116. The remainder theorem and the factor theorem	281
117. Synthetic division . . . . .	284
118. The depressed equation. . . . .	287
119. Factorization of polynomials. The number of roots of an equation. . . . .	289
*120. Proof of the uniqueness of factorization. . . . .	293
121. Imaginary roots of equations with real coefficients .	294
122. Rational roots of equations with integral coefficients	297
123. Graphical solution of equations . . . . .	302
124. Transformation reversing the signs of the roots	303
125. Upper and lower bounds for the roots. . . . .	305
126. Descartes' rule of signs . . . . .	307
127. Real roots by successive approximations . . . . .	312
128. Linear interpolation. . . . .	315
129. Transformations of equations . . . . .	316
130. Horner's method. . . . .	320
131. Relations between coefficients and roots. . . . .	324
132. Algebraic solution of equations. . . . .	326
*133. Algebraic solution of the general cubic equation .	327
*134. Algebraic solution of the general quartic equation	329

**XX. DETERMINANTS AND ELIMINATION THEORY** 332

135. Solution of linear systems in two unknowns by determinants . . . . .	332
136. Determinants of third order. . . . .	334
137. Solution of linear equations in three unknowns by determinants . . . . .	337
*138. Area of a triangle by determinants . . . . .	339
139. Determinants of order n. . . . .	341
140. Some properties of determinants. . . . .	344
141. Expansion by minors. . . . .	349
142. Solution of linear systems in n unknowns by determinants. . . . .	354
143. Systems of homogeneous linear equations . . . . .	357
144. Systems of m linear equations in n unknowns . . . . .	359
*145. The resultant of two polynomials. . . . .	362
*146. Systems of higher degree in two unknowns . . . . .	364

<b>XXI.</b>	<b>PROGRESSIONS</b>	<b>367</b>
1	147. Arithmetic progressions. . . . .	367
	148. Geometric progressions . . . . .	370
	149. Harmonic progressions . . . . .	373
	150. Infinite geometric series. Periodic decimals . . .	374
	*151. Differences of rth order. . . . .	376
	*152. Arithmetic progressions of rth order . . . . .	378
<b>XXII.</b>	<b>INEQUALITIES</b>	<b>382</b>
	153. Inequalities . . . . .	382
	154. Properties of inequalities . . . . .	383
	155. Solution of inequalities . . . . .	384
	156. Graphical significance of inequalities . . . . .	386
	*157. Linear programming . . . . .	387
<b>XXIII.</b>	<b>LOGARITHMS</b>	<b>391</b>
	158. Definition of logarithm . . . . .	391
	159. Properties of logarithms. . . . .	392
	160. Common logarithms . . . . .	394
	161. Interpolation by proportional parts. . . . .	398
	162. Significant figures . . . . .	399
	163. Computation with logarithms . . . . .	401
	*164. Cologarithms . . . . .	405
	165. Natural logarithms. Computation of tables . . . .	406
	166. Exponential and logarithmic functions and equations	407
<b>XXIV.</b>	<b>MATHEMATICS OF INVESTMENT</b>	<b>410</b>
	167. Simple interest and simple discount . . . . .	410
	168. Compound interest and compound discount. . .	412
	169. Effective and nominal rates . . . . .	416
	170. Annuities . . . . .	417
<b>XXV.</b>	<b>THE EUCLIDEAN ALGORITHM</b>	<b>421</b>
	171. The Euclidean algorithm for positive integers . . .	421
	172. Euclidean algorithm for polynomials . . . . .	424
<b>XXVI.</b>	<b>PARTIAL FRACTIONS</b>	<b>427</b>
	173. The theorem and its use. . . . .	427
	174. Proof of the theorem . . . . .	434

**XXVII] INFINITE SERIES** 436

175. Introduction. . . . .	436
176. Limit of an infinite sequence. . . . .	437
177. Convergence of infinite series . . . . .	439
178. The comparison tests. . . . .	441
*179] The decimal expression for a real number. . . . .	444
180. The ratio tests. . . . .	446
181. Series whose terms are all negative. . . . .	448
182. Alternating series. . . . .	448
183. Absolute convergence. . . . .	449
184. The general ratio tests . . . . .	450
185. Power series. . . . .	451
186. The use of series in computation. . . . .	453

**XXVIII] INTERPOLATION AND CURVE FITTING** 455

187. Linear interpolation . . . . .	455
188. Lagrange's interpolation formula. . . . .	457
189. Method of undetermined coefficients . . . . .	459
190. The Gregory-Newton interpolation formula . . . . .	460
191. Curve fitting and empirical equations. . . . .	464
192. The method of differences. . . . .	466
193. The method of averages. . . . .	469
194. The method of least squares. . . . .	473
195. Exponential and power functions. . . . .	477
196. Extrapolation . . . . .	481

**TABLES** 483

I. Powers, Roots, Reciprocals. . . . .	483-484
II. Trigonometric Functions.. . . .	485
III. Four-place Logarithm Tables. . . . .	486-487
IV. Compound Interest. . . . .	488
V. Compound Discount . . . . .	489
VI. Amount of an Annuity . . . . .	490
VII. Present Value of an Annuity. . . . .	491
VIII. Commissioners Standard Ordinary Mortality Table (1941). 492	

**ANSWERS TO ODD-NUMBERED EXERCISES** 493

**INDEX** 519