CONTENTS

Preface vi

Chapter 1. Scalars 1

1. Double addition
2. Half double addition
3. Exponentiation
4. Complex numbers
5. Affine transformations
6. Matrix multiplication
7. Modular multiplication
8. Small operations
9. Identity elements
10. Complex inverses
11. Affine inverses
12. Matrix inverses
13. Abelian groups
14. Groups
15. Independent group axioms
16. Fields
17. Addition and multiplication in fields
18. Distributive failure
19. Finite fields

Chapter 2. Vectors 17

20. Vector spaces
21. Examples
22. Linear combinations
23. Subspaces
24. Unions of subspaces

25. Spans
26. Equalities of spans
27. Some special spans
28. Sums of subspaces
29. Distributive subspaces
30. Total sets
3 1. Dependence
32. Independence

Chapter 3. Bases 39

33. Exchanging bases
34. Simultaneous complements
35. Examples of independence
36. Independence over $\mathbb{R}$ and $\mathbb{Q}$
37. Independence in $\mathbb{C}^2$
38. Vectors common to different bases
39. Bases in $\mathbb{C}^3$
40. Maximal independent sets
41. Complex as real
42. Subspaces of full dimension
43. Extended bases
44. Finite-dimensional subspaces
45. Minimal total sets
46. Existence of minimal total sets
47. Infinitely total sets
48. Relatively independent sets
49. Number of bases in a finite vector space
50. Direct sums
51. Quotient spaces
52. Dimension of a quotient space
53. Additivity of dimension

Chapter 4. Transformations 51

54. Linear transformations
55. Domain and range
56. Kernel
57. Composition
58. Range inclusion and factorization
59. Transformations as vectors
60. Invertibility
61. Invertibility examples
62. Determinants: 2 x 2

63. Determinants: n x n
64. Zero-one matrices
65. Invertible matrix bases
66. Finite-dimensional invertibility
67. Matrices
68. Diagonal matrices
69. Universal commutativity
70. Invariance
71. Invariant complements
72. Projections
73. Sums of projections
74. not quite idempotence

Chapter 5. Duality 85

75. Linear functionals
76. Dual spaces
77. Solution of equations
78. Reflexivity
79. Annihilators
80. Double annihilators
81. Adjoints
82. Adjoints of projections
83. Matrices of adjoints

Chapter 6. Similarity 97

84. Change of basis: vectors
85. Change of basis: coordinates
86. Similarity: transformations
87. Similarity: matrices
88. Inherited similarity
89. Similarity: real and complex
90. Rank and nullity
91. Similarity and rank
92. Similarity of transposes
93. Ranks of sums
94. Ranks of products
95. Nullities of sums and products
96. Some similarities
97. Equivalence
98. Rank and equivalence

Chapter 7. Canonical Forms 107
99. Eigenvalues
100. Sums and products of eigenvalues
101. Eigenvalues of products
102. Polynomials in eigenvalues
103. Diagonalizing permutations
104. Polynomials in eigenvalues, converse
105. Multiplicities
106. Distinct eigenvalues
107. Comparison of multiplicities
108. Triangularization
109. Complexification
110. Unipotent transformation
111. Nipotence
112. Nilpotent products
113. Nilpotent direct sums
114. Jordan form
115. Minimal polynomials
116. Non-commutative Lagrange interpolation

Chapter 8. Inner Product Spaces 129
117. Inner products
118. Polarization
119. The Pythagorean theorem
120. The parallelogram law
121. Complete orthonormal sets
122. Schwarz inequality
123. Orthogonal complements
124. More linear functionals
125. Adjoints on inner product spaces
126. Quadratic forms
127. Vanishing quadratic forms
128. Hermitian transformations
129. Skew transformations
130. Real Hermitian forms
131. Positive transformations
132. positive inverses
133. Perpendicular projections
134. Projections on $\mathbb{C} \times \mathbb{C}$
135. Projection order
136. Orthogonal projections

137. Hermitian eigenvalues
138. Distinct eigenvalues

Chapter 9. Normality 149
139. Unitary transformations
140. Unitary matrices
141. Unitary involutions
142. Unitary triangles
143. Hermitian diagonalization
144. Square roots
145. Polar decomposition
146. Normal transformations
147. Normal diagonalizability
148. Normal commutativity
149. Adjoint commutativity
150. Adjoint intertwining
151. Normal products
152. Functions of transformations
153. Gramians
154. Monotone functions
155. Reducing ranges and kernels
156. Truncated shifts
157. Non-positive square roots
158. Similar normal transformations
159. Unitary equivalence of transposes
160. Unitary and orthogonal equivalence
161. Null convergent powers
162. Power boundedness
163. Reduction and index 2
164. Nilpotence and reduction

Hints 169

Solutions:

Chapter 1 185
Chapter 2 204
Chapter 3 216
Chapter 4 228
Chapter 5 252
Chapter 6 259
Chapter 7 277
Chapter 8 296
Chapter 9 310