CONTENTS

PREFACE ix

CHAPTER 1: FUNDAMENTAL CONCEPTS 1

- 1.1 Sets 1
- 1.2 Functions 12
- 1.3 Relations and Equivalence Relations 25
- 1.4 Operations on Sets 26
- 1.5 Mathematical Systems Considered in This Book 30
- 1.6 References and Notes 31 References 32

CHAPTER 2: ALGEBRAIC STRUCTURES 33

- 2.1 Some Basic Structures of Algebra 34
 - A. Semigroups and Groups 36
 - B. Rings and Fields 46
 - C. Modules, Vector Spaces, and Algebras 53
 - D. Overview 61
- 2.2 Homomorphisms 62
- 2.3 Application to Polynomials 69
- 2.4 References and Notes 74 References 74

Contents

85

CHAPTER 3: VECTOR SPACES AND LINEAR TRANSFORMATIONS 75

- 3.1 Linear Spaces 75
- 3.2 Linear Subspaces and Direct Sums 81
- 3.3 Linear Independence, Bases, and Dimension
- 3.4 Linear Transformations 95
- 3.5 Linear Functionals 109
- 3.6 Bilinear Functionals 113
- 3.7 Projections 119
- 3.8 Notes and References 123 References 123

CHAPTER 4: FINITE-DIMENSIONAL VECTOR SPACES AND MATRICES 124

4.1 Coordinate Representation of Vectors 124 42 Matrices 129 A. Representation of Linear Transformations by Matrices 129 Β. Rank of a Matrix 134 C. Properties of Matrices 136 Equivalence and Similarity 4.3 148 4.4 Determinants of Matrices 155 4.5 **Eigenvalues and Eigenvectors** 163 4.6 Some Canonical Forms of Matrices 169 47 Minimal Polynomials, Nilpotent Operators and the Jordan Canonical Form 178 A. Minimal Polynomials 178 B. Nilpotent Operators 185 C. The Jordan Canonical Form 190 Bilinear Functionals and Congruence 4.8 194 4.9 Euclidean Vector Spaces 202 A. Euclidean Spaces: Definition and Properties 202 B. Orthogonal Bases 209 4.10 Linear Transformations on Euclidean Vector Spaces 216 A. Orthogonal Transformations 216 B. Adjoint Transformations 218 C. Self-Adjoint Transformations 221 D. Some Examples 227 Further Properties of Orthogonal E. Transformations 231

Contents

- 4.11 Applications to Ordinary Differential Equations 238
 A. Initial-Value Problem : Definition 238
 B. Initial-Value Problem : Linear Systems 244
- 4.12 Notes and References 261 References 262

CHAPTER 5: METRIC SPACES 263

- 5.1 Definition of Metric Spaces 264
- 5.2 Some Inequalities 268
- 5.3 Examples of Important Metric Spaces 271
- 5.4 Open and Closed Sets 275
- 5.5 Complete Metric Spaces 286
- 5.6 Compactness 298
- 5.7 Continuous Functions 307
- 5.8 Some Important Results in Applications 314
- 5.9 Equivalent and Homeomorphic Metric Spaces. Topological Spaces 317
- 5.10 Applications 323
 - A. Applications of the Contraction Mapping Principle 323
 - B.' Further Applications to Ordinary Differential Equations 329
- 5.11 References and Notes 341 References 341

CHAPTER 6: NORMED SPACES AND INNER PRODUCT SPACES 343

- 6.1 Normed Linear Spaces 344
- 6.2 Linear Subspaces 348
- 6.3 Infinite Series 350
- 6.4 Convex Sets 351
- 6.5 Linear Functionals 355
- 6.6 Finite-Dimensional Spaces 360
- 6.7 Geometric Aspects of Linear Functionals 363
- 6.8 Extension of Linear Functionals 367
- 6.9 Dual Space and Second Dual Space 370
- 6.10 Weak Convergence 372
- 6.11 Inner Product Spaces 375
- 6.12 Orthogonal Complements 381

- 6.13 Fourier Series 387
- 6.14 The Riesz Representation Theorem 393
- 6.15 Some Applications 394
 - A. Approximation of Elements in Hilbert Space (Normal Equations) 395
 - B. Random Variables 397
 - C. Estimation of Random Variables 398
- 6.16 Notes and References 404 References 404

CHAPTER 7: LINEAR OPERATORS 406

- 7.1 Bounded Linear Transformations 407
- 7.2 Inverses 415
- 7.3 Conjugate and Adjoint Operators 419
- 7.4 Hermitian Operators 427
- 7.5 Other Linear Operators: Normal Operators, Projections, Unitary Operators, and Isometric Operators 431
- 7.6 The Spectrum of an Operator 439
- 7.7 Completely Continuous Operators 447
- 7.8 The Spectral Theorem for Completely Continuous Normal Operators 454
- 7.9 Differentiation of Operators 458
- 7.10 Some Applications 465
 - A. Applications to Integral Equations 465
 - B. An Example from Optimal Control 468
 - C. Minimization of Functionals Method of Steepest Descent 471
- 7.11 References and Notes 473 References 473 Index 475