
Contents

Preface to the Second Edition	xiii
Preface to the First Edition	xv

PART 1
THEORY

CHAPTER 1	
Normed Vector Spaces	1
1.1 Introduction	1
1.2 Vector Spaces	1
1.3 Linear Independence, Basis, Dimension	7
1.4 Normed Spaces	8
1.5 Banach Spaces	16
1.6 Linear Mappings	20
1.7 Completion of Normed Spaces	27
1.8 Contraction Mappings and the Banach Fixed Point Theorem	28
1.9 Exercises	31

CHAPTER 2	
The Lebesgue Integral	37
2.1 Introduction	37
2.2 Step Functions	38
2.3 Lebesgue Integrable Functions	43
2.4 The Absolute Value of an Integrable Function	46
2.5 Series of Integrable Functions	49
2.6 Norm in $L^1(\mathbb{R})$	51
2.7 Convergence Almost Everywhere	53
2.8 Fundamental Theorems	57
2.9 Locally Integrable Functions	63
2.10 The Lebesgue Integral and the Riemann Integral	63
2.11 Lebesgue Measure on \mathbb{R}	66
2.12 Complex-Valued Lebesgue Integrable Functions	70
2.13 The Space $L^2(\mathbb{R})$	72
2.14 The Spaces $L^1(\mathbb{R}^N)$ and $L^2(\mathbb{R}^N)$	73
2.15 Convolution	77
2.16 Exercises	80
CHAPTER 3	
Hilbert Spaces and Orthonormal Systems	87
3.1 Introduction	87
3.2 Inner Product Spaces-Definition and Examples	87
3.3 Norm in an Inner Product Space	89
3.4 Hilbert Spaces-Definition and Examples	92
3.5 Strong and Weak Convergence	98
3.6 Orthogonal and Orthonormal Systems	100
3.7 Properties of Orthonormal Systems	105
3.8 Trigonometric Fourier Series	115
3.9 Orthogonal Complements and Projection Theorem	120
3.10 Linear Functionals and the Riesz Representation Theorem	125
3.11 Separable Hilbert Spaces	127
3.12 Exercises	130
CHAPTER 4	
Linear Operators on Hilbert Spaces	139
4.1 Introduction	139
4.2 Examples of Operators	140

4.3 Bilinear Functionals and Quadratic Forms	144
4.4 Adjoint and Self-Adjoint Operators	151
4.5 Invertible, Normal, Isometric, and Unitary Operators	157
4.6 Positive Operators	162
4.7 Projection Operators	168
4.8 Compact Operators	172
4.9 Eigenvalues and Eigenvectors	178
4.10 Spectral Decomposition	188
4.11 The Fourier Transform	193
4.12 Unbounded Operators	206
4.13 Exercises	214

PART 2

APPLICATIONS

CHAPTER 5

Applications to Integral and Differential Equations	223
5.1 Introduction	223
5.2 Basic Existence Theorems	224
5.3 Fredholm Integral Equations	230
5.4 Method of Successive Approximations	233
5.5 Volterra Integral Equations	235
5.6 Method of Solution for a Separable Kernel	239
5.7 Volterra Integral Equations of the First Kind and Abel's Integral Equation	243
5.8 Ordinary Differential Equations and Differential Operators	245
5.9 Sturm-Liouville Systems	254
5.10 Inverse Differential Operators and Green's Functions	259
5.11 Applications of the Fourier Transform to Ordinary Differential Equations and Integral Equations	264
5.12 Exercises	272

Chapter 6

Generalized Functions and Partial Differential Equations	279
6.1 Introduction	279
6.2 Distributions	279

6.3	Fundamental Solutions and Green's Functions for Partial Differential Equations	291
6.4	Weak Solutions of Elliptic Boundary Value Problems	311
6.5	Examples of Applications of Fourier Transforms to Partial Differential Equations	317
6.6	Exercises	330
CHAPTER 7		
	Mathematical Foundations of Quantum Mechanics	337
7.1	Introduction	337
7.2	Basic Concepts and Equations of Classical Mechanics	337
7.3	Basic Concepts and Postulates of Quantum Mechanics	349
7.4	The Heisenberg Uncertainty Principle	363
7.5	The Schrödinger Equation of Motion	365
7.6	The Schrödinger Picture	380
7.7	The Heisenberg Picture and the Heisenberg Equation of Motion	387
7.8	The Interaction Picture	391
7.9	The Linear Harmonic Oscillator	392
7.10	Angular Momentum Operators	398
7.11	The Dirac Relativistic Wave Equation	405
7.12	Exercises	409
CHAPTER 8		
	Wavelets	417
8.1	Brief Historical Remarks.	417
8.2	Continuous Wavelet Transforms	420
8.3	The Discrete Wavelet Transform	427
8.4	Multiresolution Analysis and Orthonormal Bases of Wavelets	434
8.5	Exercises	443
CHAPTER 9		
	Optimization Problems and Other Miscellaneous Applications ...	447
9.1	Introduction	447
9.2	The Gateaux and Fréchet Differentials	448
9.3	Optimization Problems and the Euler-Lagrange Equations ..	460
9.4	Minimization of Quadratic Functionals	475

9.5 Variational Inequalities	477
9.6 Optimal Control Problems for Dynamical Systems	480
9.7 Approximation Theory	487
9.8 The Shannon Sampling Theorem	492
9.9 Linear and Nonlinear Stability	496
9.10 Bifurcation Theory	500
9.11 Exercises	506
Hints and Answers to Selected Exercises	515
Bibliography	531
List of Symbols	537
Index	541