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... that he left to the modern scientific and engineering communities. The significance and generality of his profound analysis has created an extraordinarily strong impulse to greatly motivate his successors in the investigation and exploration of nonlinear dynamical systems, not to mention his influence in many other areas of engineering and mathematics (see, for example, Broecker [1983]).

For two-dimensional ODE systems, the earlier conjecture about the existence of periodic solutions given by Poincaré was formally presented by the Soviet mathematician A. A. Andronov and his colleagues [Andronov and Chaikin, 1949; Andronov et al., 1966, 1973]. Ever since then, this existence result on periodic solutions of two-dimensional ODEs was referred to as the Poincaré-Andronov conjecture in the literature. Independently, the German mathematician E. Hopf published a result with some very simple statements, proving the existence of limit cycles in an n -dimensional ODE system, for $n \geq 2$, assuming only the smoothness of the nonlinear vector field of the system [Hopf, 1943]. This is the celebrated Hopf Theorem. Basically, the theorem shows that the amplitude and frequency of the periodic solution of the equation can be calculated approximately when a key real parameter of the equation varies. In addition, the theorem shows how the stability of the periodic solution *emerging* (*bifurcating*) from the equilibrium is determined as the key parameter varies. For this reason, the result is also called the Hopf bifurcation theorem. This important result was rediscovered and applied to various areas about thirty years later by many other researchers from different disciplinary fields.

All the aforementioned works use the state-space formulation, namely, a system of n th-order ordinary differential equations. This will be referred