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The programs, in particular the key computational and graphics subroutines that appear repeatedly in the sequel, are explained in some detail in this chapter, which should be read if only for this reason. Chapter 2 provides a descriptive introduction to stochastic calculus and stochastic differential equations, including stochastic Taylor expansions and their constituent multiple stochastic integrals from which the higher order numerical schemes for stochastic differential equations are derived. The analytical solution of initial value problems for differential equations for the case of linear stochastic equations is derived in Chapter 3, first for ordinary differential equations and then for stochastic differential equations. Computing exercises for differential equations with known explicit solutions are used to illustrate theoretical convergence rates and the effects of round-off error and numerical instability. For stochastic differential equations a fundamental distinction is made between strong, pathwise approximations and weak, distributional approximations. Different requirements for each are discussed in the context of the Euler scheme applied to a simple linear stochastic differential equation. Chapters 4 and 5 are devoted to higher order schemes for strong and weak approximations, respectively. Comparative numerical studies are presented there, along with many practical hints based on the experience of the co-authors and their co-workers over the past 15 years. The final chapter, Chapter 6, introduces the reader to the vast range of situations in which the numerical simulation of stochastic differential equations can play a useful role, including stability and bifurcation in stochastic dynamical systems, filtering, testing parametric estimators, calculating invariant measures, and finance modelling. In each case several applied simulation projects are formulated as Exercises for the reader.

The diskette contains Borland TURBO PASCAL programs for all of the Problems in Chapters 1 to 5. They are based on programs and subroutines prepared by the co-authors and their co-workers, who are not professional programmers, over the past five years and are made available here solely for the

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