CONTENTS

I. Introduction and experimental results 4

A. Structure and phase diagram of the cuprates 6
1. $La_{2-x}Sr_xCuO_4$ 8
2. $YBa_2Cu_3O_{6+x}$ 9
3. $Nd_{2-x}Ce_xCuO_4$ 10

B. Normal state properties 11

C. Electronic models 12
1. Three band model 12
2. One band models 13

II. Algorithms 15

A. Lanczos technique 16
1. Method 16
2. Dynamical properties 19

B. Quantum Monte Carlo technique 21

C. Sign problem 23

III. Correlated electrons at low hole doping . . . . . 2'5

A. Results at half-filling 25

B. Properties of holes in antiferromagnets 28
1. String picture 29
2. Energy and momentum of a hole 31
3. Dispersion relation of a hole 33
4. Dynamical properties of one hole 3 4
5. Binding of holes 36
6. Quasiparticles in models of correlated electrons 38

IV. Comparing experiments with computer simulation results 41

A. Magnetic properties in the presence of *carriers* 4 1
1. Magnetic susceptibility 41
2. Antiferromagnetism **at** finite doping' 42
3. Incommensurate spin order in doped **materials** 44

B. Optical conductivity 45
1. Experimental results 46
2. Theoretical analysis of $\sigma(\omega)$ 48
3. Numerical results 51

C. Electron spectroscopy 55
1. Density of states (experiments) 56
2. Density of states (theory) 58
3. Angie-resolved photoemission 62
4. Fermi **surface** in models of **correlated** electrons 64

D. Phase separation 6.5
1. Experimental results 65
2. Theoretical results . . . . . ,5

E. Superconductivity in models of strongly correlated electrons 68
1. Superconductivity in the one and three band Hubbard models 68
2. Superconductivity in the t —J model 69

3. Phase diagram of the two dimensional t − J model 72
4. Meissner effect and flux quantization 73
5. $d_{x^2-y^2}$ superconductivity 75

V. Conclusions 76

VI. Acknowledgments 78

Footnotes 79

References 81

Figure Captions 95