

Contents

Part I. Development of the formalism

1. Introduction	3
1.1 Perturbation theory and quasi-particles	3
1.2 A brief history of bosonization in $d > 1$	5
1.3 The scope of this book	8
1.4 Notations and assumptions.	9
2. Fermions and the Fermi surface	11
2.1 The generic many-body Hamiltonian	11
2.2 The single-particle Green's function	14
2.2.1 Definition of the Green's function	14
2.2.2 Definition of the interacting Fermi surface	15
2.2.3 Landau Fermi liquids	16
2.3 The density-density correlation function	20
2.4 Patching the Fermi surface.	22
2.4.1 Definition of the patches and boxes	22
2.4.2 Linearization of the energy dispersion	25
2.4.3 Around-the-corner processes and the proper choice of the cutoffs	27
2.5 Curved patches and reduction of the patch number	28
2.6 Summary and outlook	31
3. Hubbard-Stratonovich transformations	33
3.1 Grassmannian functional integrals	33
3.2 The first Hubbard-Stratonovich transformation	36
3.2.1 Decoupling of the interaction	36
3.2.2 Transformation of the single-particle Green's function ..	37
3.3 The second Hubbard-Stratonovich transformation	40
3.3.1 Transformation of the density-density correlation function	40
3.3.2 Definition of the bosonized kinetic energy	43
3.4 Summary and outlook.	43

4. Bosonization of the Hamiltonian and the density-density correlation function	45
4.1 The generalized closed loop theorem	46
4.2 The Gaussian approximation	51
4.2.1 The effective action for the ϕ^α -field 	51
4.2.2 The Gaussian propagator of the ϕ^α -field	53
4.2.3 The effective action for the $\tilde{\rho}^\alpha$ -field 	54
4.2.4 The Gaussian propagator of the $\tilde{\rho}^\alpha$ -field 	55
4.2.5 The bosonized Hamiltonian	56
4.3 Beyond the Gaussian approximation.	58
4.3.1 General expansion of the bosonized kinetic energy	58
4.3.2 The leading correction to the effective action	61
4.3.3 The leading correction to the bosonic propagator	63
4.3.4 The hidden small parameter	65
4.3.5 Calculating corrections to the RPA via bosonization ...	69
4.4 Summary and outlook	71
5. The single-particle Green's function.	73
5.1 The Gaussian approximation with linearized energy dispersion	73
5.1.1 The Green's function for fixed background field.	74
5.1.2 Gaussian averaging: calculation of the Debye-Waller ..	80
5.1.3 The Green's function in real space	82
5.1.4 The underlying asymptotic Ward identity	83
5.1.5 The Fermi liquid renormalization factors Z^α and Z_m^α ..	88
5.2 Beyond the Gaussian approximation.	92
5.2.1 The Green's function for fixed background field.	94
5.2.2 Non-Gaussian averaging	100
5.3 The Gaussian approximation with non-linear energy dispersion	105
5.3.1 The average eikonal.	105
5.3.2 The prefactor Green's functions	106
5.3.3 Connection with lowest order perturbation theory.	110
5.4 Summary and outlook	114

Part II. Applications to physical systems

6. Singular interactions ($f_q \sim q ^{-\eta}$)	121
6.1 Manipulations with the help of the dynamic structure factor ..	122
6.1.1 Non-linear energy dispersion	122.
6.1.2 The limit of linear energy dispersion	123
6.1.3 Finite versus infinite patch number	125
6.2 The static Debye-Waller factor for linearized energy dispersion	127
6.2.1 Consequences of spherical symmetry	127
6.2.2 The existence of the quasi-particle residue	131
6.2.3 Why the Coulomb interaction is so nice	132

6.2.4	The sub-leading corrections for $0 < \eta < 2(d-1)$	133
6.2.5	The regime $\eta \geq 2(d-1)$	135
6.3	Luttinger liquid behavior in $d = 1$	137
6.4	Summary and outlook.	141
7.	Quasi-one-dimensional metals	143
7.1	The Coulomb interaction in chains without interchain hopping	144
7.2	Finite interchain hopping	149
7.2.1	The 4-patch model.	150
7.2.2	How curvature kills the nesting singularity	158
7.2.3	Anomalous scaling in a Fermi liquid	160
7.2.4	The nesting singularity for general Fermi surfaces.	161
7.3	Summary and outlook	163
8.	Electron-phonon interactions	165
8.1	The effective interaction	166
8.1.1	The Debye model	166
8.1.2	Integration over the phonons	169
8.2	The Debye-Waller factor	171
8.3	Phonon energy shift and phonon damping	172
8.4	The quasi-particle residue	175
8.4.1	Isotropic phonon dispersion	176
8.4.2	Quasi-one-dimensional electrons or phonons	177
8.5	Summary and outlook	179
9.	Fermions in a stochastic medium	181
9.1	The average Green's function	182
9.1.1	Non-interacting disordered fermions	182
9.1.2	Interacting disordered fermions	185
9.2	Static disorder	187
9.3	Dynamic disorder.	189
9.3.1	Gaussian white noise.	190
9.3.2	Finite correlation time	193
9.4	Summary and outlook	194
10.	Transverse gauge fields	197
10.1	Effective actions	200
10.1.1	The coupled matter gauge field action	200
10.1.2	The effective matter action.	203
10.1.3	The effective gauge field action	205
10.2	The Green's function in Gaussian approximation.	207
10.2.1	The Green's function for fixed gauge field	208
10.2.2	Gaussian averaging	209
10.3	Transverse screening	213
10.3.1	The transverse dielectric tensor	213

10.3.2 Screening and gauge invariance.	214
10.3.3 The transverse dielectric function for spherical Fermi surfaces	216
10.4 The transverse Debye-Waller factor	218
10.4.1 Exact rescalings..	218
10.4.2 The relevance of curvature	221
10.4.3 Two-dimensional Maxwell-Chern-Simons theory	224
10.5 Summary and outlook	231
Appendix: Screening and collective modes	235
A.1 The non-interacting polarization for spherical Fermi surfaces .	235
A.2 The dynamic structure factor for spherical Fermi surfaces	237
A.3 Collective modes for singular interactions	240
A.3.1 <u>The Coulomb interaction in $1 \leq d \leq 3$</u>	240
A.3.2 General singular interactions	242
A.4 Collective modes for finite patch number.	243
References.....	247
Index.....	256