

# CONTENTS

Foreword . . . . . * . . . . *	v
<b>DIRECTIONS IN CLASSICAL CHAOS</b>	1
<i>Joseph Ford</i>	
1. Back to the Future . . . . .	1
2. Generalized Uncertainty Principle . . . . .	3
3. The Uses of Chaos . . . . .	5
4. Algorithmic Integrability . . . . .	8
5. Thermodynamic Law . . . . .	10
6. Theme and Variations . . . . .	12
<b>NONLINEAR RESONANCE AND CHAOS IN CONSERVATIVE SYSTEMS</b>	17
<i>L. E. Reichl &amp; W. M. Zheng</i>	
1. Introduction . . . . .	17
2. Important Properties of Hamiltonian Systems . . . . .	21
3. Unperturbed Double Well and Pendulum Systems . . . . .	26
3.1 Quartic double well system . . . . .	26
3.2 The pendulum . . . . .	32
4. Resonance Structure of the Duffing System . . . . .	34
5. Two Resonance Hamiltonian . . . . .	41
6. Behavior near the Separatrix — Melnikov Integral . . . . .	51
7. Whisker Map . . . . .	58
8. Standard Map . . . . .	64
9. Cantori . . . . .	73
<i>Appendix A: Elliptic Functions and Integrals . . . . .</i>	<i>79</i>
<i>Appendix B: Renormalization Group Transformation . . . . .</i>	<i>81</i>
<i>Appendix C: Evaluation of Melnikov Integral . . . . .</i>	<i>88</i>

GENERALIZED RENORMALIZATION GROUP ANALYSIS OF PERIOD-DOUBLING BIFURCATIONS	91
<b><i>K. L. Liu &amp; K. Young</i></b>	
1. Introduction . . . . .	91
2. Renormalization Group . . . . .	94
3. Truncation . . . . .	97
3.1 Analytic solution of <b>(3, 3)</b> order . . . . .	97
3.2 Results for higher orders . . . . .	99
4. Functional Relations . . . . .	101
4.1 Functional derivation . . . . .	101
4.2 “Experimental” derivation . . . . .	104
4.3 Discussion . . . . .	105
5. Other Sequences . . . . .	107
5.1 Subasymptotic eigenvalues . . . . .	107
5.2 Generalized RG . . . . .	108
5.3 Proof of RG properties . . . . .	109
5.3.1 Upsequence . . . . .	111
5.3.2 Down sequence . . . . .	111
5.3.3 The eigenvalues . . . . .	111
5.3.4 RG transformation . . . . .	112
5.3.5 Relation to $\psi$ . . . . .	115
5.3.6 Other sequences . . . . .	116
5.3.7 Discussion . . . . .	117
6. Conclusion . . . . .	117
<b>Appendix A:</b> Derivation of (29) . . . . .	118
<b>Appendix B:</b> . . . . .	119
APPLICATION OF DIMENSION ALGORITHMS TO EXPERIMENTAL CHAOS	122
<b><i>Gottfried Mayer-Kress</i></b>	
1. Introduction . . . . .	122
2. Basic Concepts . . . . .	123
2.1 Geometrical reconstruction . . . . .	123
2.2 Gauge functions . . . . .	125
2.2.1 Pointwise dimension . . . . .	126
2.2.2 The “correlation -1 or “Grassberger-Procaccia” dimension . . . . .	127

2.2.3 Information- (mass) dimension . . . . . 129

2.2.4 Averaged pointwise dimension . . . . . 129

2.2.5 Average distances of nearest neighbors . . . . . 130

3. Reliability and Stability of the Algorithms . . . . . 13 1

3.1 Minimal number of data points and sampling rate of  
the signal . . . . . 13 1

3.2 Filtering of the signals . . . . . , 132

3.3 Geometrical effects . . . . . 134

3.4 Random walks . . . . . 135

EMERGENCE OF CHAOS IN LASER SYSTEMS AND  
THE DEVELOPMENT OF DIAGNOSTIC TECHNIQUES 148

***J. R. Tredicce & L. M. Narducci***

1. Introduction . . . . . 148

2. Classification of Lasers . . . . . 148

3. Chaotic Laser Systems . . . . . 15 1

4. Diagnostic Techniques . . . . . 158

5. Conclusions . . . . . 161

DISSIPATIVE CLASSICAL AND QUANTUM DYNAMICS:  
THE MORSE OSCILLATOR 164

***Jian -Min Yuan***

1. Introduction . . . . . 164

2. Classical Model . . . . . 167

3. Semiclassical Model . . . . . 175

4. Quantum Mechanical Study . . . . . 184

5. Experimental Verification of Bistable and Chaotic Behavior . . . . . 196

6. Discussions of Remaining Questions . . . . . 198

7. Summary . . . . . 199

TRANSITIONS TO CHAOS IN HIGHER DIMENSIONS 206

***Bambi Hu & Jian-Min Mao***

1. Introduction . . . . . 206

2. Period Doubling in Four -Dimensional  
Volume-Preserving Maps . . . . . 207

2.1	Introduction	208
2.1.1	Canonical transformation and symplecticity	208
2.1.2	Stability analysis	210
2.1.3	Bifurcation point and two-parameter search	213
2.2	Search for a complete period-doubling sequence	215
2.2.1	Hénon-like four-dimensional symplectic map	215
2.2.2	A complete period-doubling sequence	216
2.2.3	Scaling factors	218
2.3	Further search for other period-doubling sequences in symmetric four-dimensional volume-preserving maps	225
2.3.1	Bifurcation of stability regions in the parameter plane	226
2.3.2	Period-doubling bifurcation routes and scaling factors	230
2.3.3	Eigenvalue – matching renormalization	236
2.4	Fixed maps of the renormalization operator	239
2.4.1	Symmetric four-dimensional volume-preserving maps	239
2.4.2	Renormalization calculation	243
2.4.3	Three fixed maps corresponding to pitchfork bifurcation	244
2.4.4	Renormalization of asymmetric volume-preserving four-dimensional maps	248
2.5	Summary	252
3.	Quasiperiodicity with Three Incommensurate Frequencies	253
3.1	Introduction	253
3.2	Formulation of the three-frequency problem	254
3.2.1	Review of the two-frequency problem	254
3.2.2	Generalization to three frequencies	258
3.3	Singular homeomorphisms on torus.	262
3.4	Numerical results	263
3.5	Concluding remarks	266

## PHENOMENOLOGY OF SPATIO-TEMPORAL CHAOS

272

*James P. Crutchfield & Kunihiko Kaneko*

1.	Complexity and Statistical Mechanics of Deterministic Behavior	272
2.	Lattice Dynamical Systems: Prototypes for Spatio-Temporal Complexity	276
2.1	Restriction to spatial lattices	276
2.2	Architectural relationship to other spatially-extended systems	278

2.3	Choice of local dynamics	280
2.4	Coupling classes	280
2.5	Boundary conditions	281
2.6	Initial conditions	281
3.	Period-Doubling Lattices	282
3.1	Dependence of domain behavior on domain size	287
3.2	Kink self-similarity	290
4.	Spatial Mode Instability: The Transition from Torus to Chaos	290
5.	Soliton Turbulence	313
6.	Intermittency Lattices	317
6.1	Spatial Pomeau-Manneville intermittency	318
6.2	Intermittency via pattern competition	322
7.	Open Flow Lattices	325
7.1	Spatial period-doubling	328
7.2	Spatial amplification of fluctuations	328
7.3	Domain walls and kink dynamics	338
8.	Transient Spatial Chaos	339
9.	Summary	343