Contents

Foreword *page* xiii
Acknowledgements xviii

1. Universal laws of nature 1
   1.1 Mechanics in the context of history 1
   1.2 The foundations of mechanics 10
   1.3 Conservation laws for N bodies 25
   1.4 The geometric invariance principles of physics 29
   1.5 Covariance of vector equations vs invariance of solutions 37
   Exercises 41

2. Lagrange's and Hamilton's equations 45
   2.1 Overview 45
   2.2 Extrema of functionals 46
   2.3 Newton's equations from the action principle 50
   2.4 Arbitrary coordinate transformations and Lagrange's equations 53
   2.5 Constrained Lagrangian systems 60
   2.6 Symmetry, invariance, and conservation laws 63
   2.7 Gauge invariance 70
   2.8 Hamilton's canonical equations 72
   Exercises 76

3. Flows in phase space 80
   3.1 Solvable vs integrable 80
   3.2 Streamline motions and Liouville's theorem 81
   3.3 Equilibria and linear stability theory 94
   3.4 One degree of freedom Hamiltonian systems 101
   3.5 Introduction to 'clockwork': stable periodic and stable quasiperiodic orbits 103
   3.6 Introduction to integrable flows 104
   3.7 Bifurcations in Hamiltonian systems 118
   Exercises 121

4. Motion in a central potential 126
   4.1 Overview 126
   4.2 Integration via two constants of the motion 127
   4.3 Maps, winding numbers, and orbital stability 129
   4.4 Clockwork is rare in mechanical systems 134
   4.5 Hidden symmetry in the Kepler problem 138

4.6 Periodicity and nonperiodicity that are not like clockwork 141
Exercises 144

5. Small oscillations about equilibria 148
5.1 Introduction 148
5.2 The normal mode transformation 148
5.3 Coupled pendulums 152
Exercises 154

6. Integrable and chaotic oscillations 156
6.1 Qualitative theory of integrable canonical systems 156
6.2 Return maps 162
6.3 A chaotic nonintegrable system 163
6.4 Area-preserving maps 170
6.5 Computation of exact chaotic orbits 175
6.6 On the nature of deterministic chaos 177
Exercises 184

7. Parameter-dependent transformations 187
7.1 Introduction 187
7.2 Phase flows as transformation groups 187
, 7.3 One-parameter groups of transformations 189
7.4 Two definitions of integrability 192
7.5 Invariance under transformations of coordinates 194
7.6 Power series solutions of systems of differential equations 195
7.7 Lie algebras of vector fields 197
Exercises 202

8. Linear transformations, rotations, and rotating frames 204
8.1 Overview 204
8.2 Similarity transformations 204
8.3 Linear transformations and eigenvalue problems 205
8.4 Rotations form a Lie group 208
8.5 Dynamics in rotating reference frames 213
Exercises 220

9. Rigid body motions 223
9.1 Euler's equations of rigid body motion 223
9.2 Euler's rigid body 228
9.3 The Euler angles 232
9.4 Lagrange's top 234
9.5 Integrable problems in rigid body dynamics 237
9.6 Noncanonical flows as iterated maps 240
9.7 Nonintegrable rigid body motions 24
Exercises 24

10. Lagrangian dynamics and transformations in configuration space 24
10.1 Invariance and covariance 24

10.2 Geometry of the motion in configuration space 251
10.3 Basis vectors in configuration space 256
10.4 Canonical momenta and covariance of Lagrange's equations in configuration space 258
Exercises 261

11. Relativity, geometry, and gravity 262
11.1 Electrodynamics 262
11.2 The principle of relativity 265
11.3 Coordinate systems and integrability on manifolds 275
11.4 Force-free motion on curved manifolds 280
11.5 Newtonian gravity as geometry in curved space-time 284
11.6 Einstein's gravitational field equations 287
11.7 Invariance principles and laws of nature 291
Exercises 294

12. Generalized vs nonholonomic coordinates 295
12.1 Group parameters as nonholonomic coordinates 295
12.2 Euler–Lagrange equations for nonintegrable velocities 302
12.3 Group parameters as generalized coordinates 304
Exercises 306

13. Noncanonical flows 307
13.1 Flows on spheres 307
13.2 Local vs complete integrability 311
13.3 Globally integrable noncanonical flows 316
13.4 Attractors 321
13.5 Damped-driven Euler–Lagrange dynamics 325
13.6 Liapunov exponents, geometry, and integrability 331
Exercises 332

14. Damped-driven Newtonian systems 334
14.1 Period doubling in physics 334
14.2 Fractal and multifractal orbits in phase space 347
14.3 Strange attractors 359
14.4 The two-frequency problem 361
Exercises 371

15. Hamiltonian dynamics and transformations in phase space 373
15.1 Generating functions for canonical transformations 373
15.2 Poisson brackets and infinitesimal transformations: symmetry, invariance, and conservation laws 377
15.3 Lie algebras of Hamiltonian flows 386
15.4 The Dirac quantization rules 390
15.5 Hidden symmetry: superintegrability of the Kepler and isotropic harmonic oscillator problems 393
15.6 Noncanonical Hamiltonian flows 397
Exercises 406

16. Integrable canonical flows 409
16.1 Solution by canonical transformations 409
16.2 The Hamilton-Jacobi equation 412
16.3 Integrability and symmetry 415
16.4 Action-angle variables 421
16.5 Central potentials 426
16.6 The action integral in quantum theory 427
Exercises 429

17. Nonintegrable canonical flows 430
17.1 The Henón–Heiles model 430
17.2 Perturbation theory and averaging 432
17.3 The standard map 435
17.4 The Kolmogorov-Arnol'd-Moser (KAM) theorem 437
17.5 Is clockwork possible in the solar system? 439
Exercises 440

18. Simulations, complexity, and laws of nature 441
18.1 Integrability, chaos, and complexity 441
18.2 Simulations and modelling without invariance 442
18.3 History in the context of mechanics 448
. Exercises 452

Bibliography 453
Index 467