

Contents

Foreword	v
Prerequisites	ix
PART ONE	
Basic Theory	1
CHAPTER I	
Complex Numbers and Functions	3
1. Definition	3
2. Polar Form	8
3. Complex Valued Functions	12
4. Limits and Compact Sets	17
Compact Sets	21
5. Complex Differentiability	27
6. The Cauchy-Riemann Equations	31
7. Angles Under Holomorphic Maps	33
CHAPTER II	
Power Series	37
1. Formal Power Series	37
2. Convergent Power Series	47
3. Relations Between Formal and Convergent Series	60
Sums and Products	60
Quotients	64
Composition of Series	66
4. Analytic Functions	68
5. Differentiation of Power Series	72

6. The Inverse and Open Mapping Theorems	76
7. The Local Maximum Modulus Principle	83
 CHAPTER III	
Cauchy's Theorem, First Part	86
1. Holomorphic Functions on Connected Sets	86
Appendix: Connectedness	92
2. Integrals Over Paths	94
3. Local Primitive for a Holomorphic Function	104
4. Another Description of the Integral Along a Path'	110
5. The Homotopy Form of Cauchy's Theorem	115
6. Existence of Global Primitives. Definition of the Logarithm	119
7. The Local Cauchy Formula	125
 CHAPTER IV	
Winding Numbers and Cauchy's Theorem	133
1. The Winding Number	134
2. The Global Cauchy Theorem	138
Dixon's Proof of Theorem 2.5 (Cauchy's Formula)	147
3. Artin's Proof	149
 CHAPTER V	
Applications of Cauchy's Integral Formula	156
1. Uniform Limits of Analytic Functions	156
2. Laurent Series	161
3. Isolated Singularities	165
Removable Singularities	165
Poles	166
Essential Singularities	168
 CHAPTER VI	
Calculus of Residues	173
1. The Residue Formula	173
Residues of Differentials	184
2. Evaluation of Definite Integrals	191
Fourier Transforms	194
Trigonometric Integrals	197
Mellin Transforms	199
 CHAPTER VII	
Conformal Mappings	208
§1. Schwarz Lemma	210
§2. Analytic Automorphisms of the Disc	212
§3. The Upper Half Plane	215
§4. Other Examples	220
§5. Fractional Linear Transformations	231

CHAPTER VIII	
Harmonic Functions	241
1. Definition	241
Application: Perpendicularity	246
Application : Flow Lines	248
2. Examples	252
3. Basic Properties of Harmonic Functions	259
4. The Poisson Formula	271
The Poisson Integral as a Convolution	273
5. Construction of Harmonic Functions	276
6. Appendix. Differentiating Under the Integral Sign	286
PART TWO	
Geometric Function Theory	291
CHAPTER IX	
Schwarz Reflection	293
1. Schwarz Reflection (by Complex Conjugation)	293
2. Reflection Across Analytic Arcs	297
3. Application of Schwarz Reflection []	303
CHAPTER XI	
The Riemann Mapping Theorem	306
1. Statement of the Theorem	306
2. Compact Sets in Function Spaces	308
3. Proof of the Riemann Mapping Theorem	311
4. Behavior at the Boundary	314
CHAPTER XI	
Analytic Continuation Along Curves	322
1. Continuation Along a Curve	322
2. The Dilogarithm	331
3. Application to Picard's Theorem	335
PART THREE	
Various Analytic Topics	337
CHAPTER XII	
Applications of the Maximum Modulus Principle and Jensen's Formula	339
1. Jensen's Formula	340
2. The Picard–Borel Theorem	346
3. Bounds by the Real Part, Borel–Carathéodory Theorem []	354
4. The Use of Three Circles and the Effect of Small Derivatives	356
Hermite Interpolation Formula	358
5. Entire Functions with Rational Values []	360
6. The Phragmen–Lindelöf and Hadamard Theorems	365

CHAPTER XIII	
Entire and Meromorphic Functions	372
1. Infinite Products	372
2. Weierstrass Products	376
3. Functions of Finite Order	382
4. Meromorphic Functions, Mittag-Leffler Theorem	387
CHAPTER XIV	
Elliptic Functions	391
1. The Liouville Theorems	391
2. The Weierstrass Function	395
3. The Addition Theorem	400
4. The Sigma and Zeta Functions	403
CHAPTER XV	
The Gamma and Zeta Functions	408
1. The Differentiation Lemma	409
2. The Gamma Function	413
Weierstrass Product	413
The Gauss Multiplication Formula (Distribution Relation)	416
The (Other) Gauss Formula	418
The Mellin Transform	420
The Stirling Formula	422
Proof of Stirling's Formula	424
3. The Lerch Formula	431
4. Zeta Functions	433
CHAPTER XVI	
The Prime Number Theorem	440
1. Basic Analytic Properties of the Zeta Function	441
2. The Main Lemma and its Application	446
3. Proof of the Main Lemma	449
Appendix	453
1. Summation by Parts and Non-Absolute Convergence	453
2. Difference Equations	457
3. Analytic Differential Equations	461
4. Fixed Points of a Fractional Linear Transformation	465
§5. Cauchy's Formula for C^∞ Functions	467
6. Cauchy's Theorem for Locally Integrable Vector Fields	472
Bibliography	479
Index	481