

Contents

Preface	V
1 Introduction	1
1.1 Cuprate superconductors	1
1.1.1 Structure	2
1.1.2 Doping	3
1.1.3 Effective mass anisotropy and spatial dimensionality	7
1.1.4 Pseudogap	10
1.1.5 Symmetry of the order parameter	13
1.1.6 Importance of critical fluctuations	15
1.2 Universal critical properties of continuous phase transitions	18
1.2.1 Static critical properties at finite temperature	18
1.2.2 Dynamic critical properties at finite temperature	23
1.2.3 Quantum critical properties	25
1.3 Finite size effect and corrections to scaling	32
2 Ginzburg - Landau phenomenology	37
2.1 London phenomenology	37
2.2 Ginzburg - Landau functional	46
2.3 Mean-field treatment	48
2.3.1 Meissner phase	49
2.3.2 Length scales: London penetration depth and correlation length	51
2.3.3 Classification of superconductors	55
2.3.4 Upper critical field	57
2.4 Flux quantization	59
2.5 London model and first flux penetration field	61
2.6 Effective mass anisotropy	64

2.6.1 3D anisotropic London model	67
3 Gaussian thermal fluctuations	73
3.1 Gaussian fluctuations around the mean field solution	73
3.2 Gaussian order parameter fluctuations	74
3.3 Gaussian vector potential fluctuations	79
3.4 Relevance of vector potential fluctuations	80
3.5 Helicity modulus	82
3.6 Effective mass anisotropy	85
3.7 Fluctuation induced diamagnetism	88
3.7.1 Isotropic system	88
3.7.2 Effective mass anisotropy	94
3.7.3 Magnetic torque	96
4 Superfluidity and the n-vector model	99
4.1 Ideal Bose gas	101
4.2 Charged Bose gas subjected to a magnetic field	109
4.3 Weakly interacting Bose gas	111
4.4 Hydrodynamic approach	114
4.5 The n-vector model	118
5 Universality and scaling theory of classical critical phenomena at finite temperature	125
5.1 Static critical phenomena in isotropic systems	125
5.2 Superconductors with effective mass anisotropy	136
5.3 Dimensional analysis	149
5.3.1 Static critical properties	149
5.3.2 Classical dynamic critical phenomena	151
5.4 Implications of the universal critical amplitude relations	153
6 Experimental evidence for classical critical behavior	157
6.1 Critical behavior close to optimum doping	157
6.1.1 Specific heat in zero field	157
6.1.2 Temperature dependence of the penetration depth	169
6.1.3 Corrections to scaling	171
6.1.4 Temperature dependence of the diamagnetic susceptibility	175
6.1.5 Scaling of the magnetization	175
6.1.6 Crossing point phenomenon	177
6.1.7 Magnetic torque and universal scaling function	181

6.1.8 Magnetic field tuned phase transitions: Melting transition	189
6.1.9 Magnetic field tuned phase transitions: Superconductor - normal conductor and insulator transitions	194
6.1.10 Evidence for a Kosterlitz - Thouless - Berezinskii transi- tion in thin films	201
6.1.11 Temperature driven 2D to 3D crossover	206
6.2 Doping dependence of the critical behavior	212
6.3 Evidence for dynamic scaling	219
6.4 Vortex glass to vortex fluid transition	220
6.5 The (H,T) phase diagram of extreme type II superconductors emerging from Monte Carlo simulations	224
7 Quantum Phase Transitions	233
7.1 Scaling theory of quantum critical phenomena	233
7.2 Quantum critical phenomena: conventional superconductors .	242
7.3 Quantum critical phenomena: cuprate superconductors . . .	248
7.3.1 Doping and disorder tuned superconductor to insulator transition	248
7.3.2 Film thickness tuned superconductor to insulator tran- sition	256
7.3.3 Doping dependence of the chemical potential	260
7.3.4 Magnetic field tuned transition	261
7.3.5 Nature of the non-superconducting phase	265
7.3.6 Superconductor to normal conductor transition	268
8 Implications	273
8.1 Interlayer tunneling model	273
8.2 Symmetry of the order parameter	276
8.3 Suppression of the transition temperature due to dimensional crossover and quantum fluctuations	277
8.4 Pseudogap features	280
8.5 Relationship between low frequency conductivity and zero tem- perature penetration depth	284
8.6 Doping and pressure dependences of critical amplitudes . . .	289
8.7 Doping dependence of isotope and pressure coefficients . . .	295
8.8 Bose gas approach	298
8.9 Effective pair mass	299
8.10 Emerging phase diagrams	301

A Mean field treatment	309
A.1 Ising Model	309
A.2 XY Model	315
B XY model	319
B.1 3D-2D Crossover in the XY model	319
B.1.1 2D-XY model	320
B.1.2 3D-XYI model	324
B.1.3 Layered XY model	327
B.1.4 Anisotropic XY model	331
B.2 Superconducting networks and films	332
B.2.1 Models	332
B.2.2 Uniform superconducting films	335
C Quantum phase transitions	337
C.1 The harmonic oscillator	337
C.2 Large-n limit of a model for distortive phase transitions	339
C.3 Onset of superfluidity in the ideal Bose gas	343
C.4 Superconductors	344
D BCS theory	351
D.1 Cooper instability	351
D.2 Electron-phonon interaction	354
D.3 Ground state in the BCS approximation	355
D.4 Thermodynamic properties in the BCS - approximation	361
D.5 Simple model	363
E Superconducting properties of the attractive Hubbard model	367
E.1 BCS — BEC crossover	367
E.2 BCS treatment of the attractive Hubbard model	379
E.3 Phase diagram of the attractive Hubbard model on a lattice	388
E.4 2D-XYI behavior and KT transition in the attractive Hubbard model	400
References	411
Index	427