

# Contents

## Preface To The Third Edition

xiii

## Glossary of Symbols

xv

## 1. Basic Descriptions and Properties

1

- 1.1. Deterministic Versus Random Data, 3
- 1.2. Classifications of Deterministic Data, 3
  - 1.2.1. Sinusoidal Periodic Data, 4
  - 1.2.2. Complex Periodic Data, 6
  - 1.2.3. Almost-Periodic Data, 7
  - 1.2.4. Transient Nonperiodic Data, 9
- 1.3. Classifications of Random Data, 10
  - 1.3.1. Stationary Random Data, 12
  - 1.3.2. Ergodic Random Data, 12
  - 1.3.3. Nonstationary Random Data, 13
  - 1.3.4. Stationary Sample Records, 14
- 1.4. Analysis of Random Data, 14
  - 1.4.1. Basic Descriptive Properties, 14
  - 1.4.2. Input/Output Relations, 20
  - 1.4.3. Error Analysis Criteria, 21
  - 1.4.4. Data Analysis Procedures, 24

## 2. Linear Physical Systems

27

- 2.1. Constant-Parameter Linear Systems, 27
- 2.2. Basic Dynamic Characteristics, 29
- 2.3. Frequency Response Functions, 31
- 2.4. Illustrations of Frequency Response Functions, 32
  - 2.4.1. Mechanical Systems, 31

- 2.4.2. Electrical Systems, 41
- 2.4.3. Other Systems, 44
- 2.5. Practical Considerations, 44

### 3. Probability Fundamentals

**48**

- 3.1. One Random Variable, 48
  - 3.3.1. Probability Density and Distribution Functions, 48
  - 3.1.2. Expected Values, 53
  - 3.1.3. Change of Variables, 53
  - 3.1.4. Moment-Generating and Characteristic Functions, 55
  - 3.1.5. Chebyshev's Inequality, 57
- 3.2. Two Random Variables, 58
  - 3.2.1. Expected Values and Correlation Coefficient, 59
  - 3.2.2. Distribution for Sum of Two Random Variables, 60
  - 3.2.3. Joint Moment Generating and Characteristic Functions, 62
- 3.3. Gaussian (Normal) Distribution, 64
  - 3.3.1. Central Limit Theorem, 65
  - 3.3.2. Joint Gaussian (Normal) Distribution, 67
  - 3.3.3. Moment-Generating and Characteristic Functions, 68
  - 3.3.4. N-Dimensional Gaussian (Normal) Distribution, 69
- 3.4. Rayleigh Distribution, 73
  - 3.4.1. Distribution of Envelope and Phase for Narrow Bandwidth Data, 73
  - 3.4.2. Distribution of Output Record for Narrow Bandwidth Date, 77
- 3.5. Higher-Order Changes of Variables, 78

### 4. Statistical Principles

**86**

- 4.1. Sample Values and Parameter Estimates, 86
- 4.2.** Important Probability Distribution Functions, 89
  - 4.2.1. Gaussian (Normal) Distribution, 89
  - 4.2.2. Chi-Square Distribution, 90
  - 4.2.3. The  $t$  Distribution, 91
  - 4.2.4. The  $F$  Distribution, 92
- 4.3.** Sampling Distributions and Illustrations, 93
  - 4.3.1. Distribution of Sample Mean with Known Variance, 93
  - 4.3.2. Distribution of Sample Variance, 94

- 4.3.3. Distribution of Sample Mean with Unknown Variance, 95
- 4.3.4. Distribution of Ratio of Two Sample Variances, 95
- 4.4. Confidence Intervals, 96
- 4.5. Hypothesis Tests, 99
  - 4.5.1. Chi-Square Goodness-of-Fit Test, 103
  - 4.5.2. Nonparametric Trend Test, 105
- 4.6. Correlation and Regression Procedures, 108
  - 4.6.1. Linear Correlation Analysis, 108
  - 4.6.2. Linear Regression Analysis, 111

## 5. Stationary Random Processes

118

- 5.1. Basic Concepts, 118
  - 5.1.1. Correlation (Covariance) Functions, 120
  - 5.1.2. Examples of Autocorrelation Functions, 122
  - 5.1.3. Correlation Coefficient Functions, 124
  - 5.1.4. Cross-Correlation Function for Time Delay, 126
- 5.2. Spectral Density Functions, 128
  - 5.2.1. Spectra via Correlation Functions, 128
  - 5.2.2. Spectra via Finite Fourier Transforms, 138
  - 5.2.3. Spectra via Filtering-Squaring-Averaging, 141
  - 5.2.4. Wave-Number Spectra, 144
  - 5.2.5. Coherence Functions, 146
  - 5.2.6. Cross-Spectrum for Time Delay, 147
  - 5.2.7. Location of Peak Value, 149
  - 5.2.8. Uncertainty Relation, 150
  - 5.2.9. Uncertainty Principle and Schwartz Inequality, 154
- 5.3. Ergodic and Gaussian Random Processes, 155
  - 5.3.1. Ergodic Random Processes, 156
  - 5.3.2. Sufficient Conditions for Ergodicity, 158
  - 5.3.3. Gaussian Random Processes, 161
  - 5.3.4. Linear Transformations of Random Processes, 163
- 5.4. Derivative Random Processes, 165
  - 5.4.1. Correlation Functions, 166
  - 5.4.2. Spectral Density Functions, 169
- 5.5. Level crossings and Peak Values, 170
  - 5.5.1. Expected Number of Level Crossings per Unit Time, 170
  - 5.5.2. Peak Probability Functions for Narrow Bandwidth Data, 174

5.5.3.	Expected Number and Spacing of Positive Peaks, 177	
5.5.4.	Peak Probability Functions for Wide Bandwidth Data, 178	
5.5.5.	Derivations. 180	
<b>6.</b>	<b>Single-Input/Output Relationships</b>	<b>189</b>
6.1.	Single-Input/Single-Output Models, 189	
6.1.1.	Correlation and Spectral Relations, 189	
6.1.2.	Ordinary Coherence Functions, 196	
6.1.3.	Models with Extraneous Noise, 200	
6.1.4.	Optimum Frequency Response Functions, 204	
6.2.	Single-Input/Multiple-Output Models, 208	
6.2.1.	Single-Input/Two-Output Model, 208	
6.2.2.	Single-Input/Multiple-Output Model, 210	
6.2.3.	Removal of Extraneous Noise, 212	
<b>7</b>	<b>Multiple-Input/Output Relationships</b>	<b>218</b>
7.1.	Multiple-Input/Single-Output Models, 218	
7.1.1.	General Relationships, 218	
7.1.2.	General Case of Arbitrary Inputs, 222	
7.1.3.	Special Case of Mutually Uncorrelated Inputs, 223	
7.2.	Two-Input/One-Output Models, 224	
7.2.1.	Basic Relationships, 224	
7.2.2.	Optimum Frequency Response Functions, 228	
7.2.3.	Ordinary and Multiple Coherence Functions, 230	
7.2.4.	Conditioned Spectral Density Functions, 232	
7.2.5.	Partial Coherence Functions, 237	
7.3.	General and Conditioned Multiple-Input Models, 240	
7.3.1.	Conditioned Fourier Transforms, 242	
7.3.2.	Conditioned Spectral Density Functions, 243	
7.3.3.	Optimum Systems for Conditioned Inputs, 244	
7.3.4.	Algorithm for Conditioned Spectra, 246	
7.3.5.	Optimum Systems for Original Inputs, 249	
7.3.6.	Partial and Multiple Coherence Functions, 252	
7.4.	Modified Procedure to Solve Multiple-Input/Single-Output Models, 254	
7.4.1.	Three-Input/Single-Output Models, 256	
7.4.2.	Formulas for Three-Input/Single-Output Models, 258	

**8. Statistical Errors in Basic Estimates**

272

- 8.1. Definition of Errors, 272
- 8.2. Mean and Mean Square Value Estimates, 276
  - 8.2.1. Mean Value Estimates, 276
  - 8.2.2. Mean Square Value Estimates, 279
  - 8.2.3. Variance Estimates, 284
- 8.3. Probability Density Function Estimates, 286
  - 8.3.1. Bias of the Estimate, 287
  - 8.3.2. Variance of the Estimate, 289
  - 8.3.3. Normalized rms Error, 290
  - 8.3.4. Joint Probability Density Function Estimates, 290
- 8.4. Correlation Function Estimates, 291
  - 8.4.1. Bandwidth-Limited Gaussian White Noise, 294
  - 8.4.2. Noise-to-Signal Considerations, 295
  - 8.4.3. Location Estimates of Peak Correlation Values, 297
- 8.5. Autospectral Density Function Estimates, 299
  - 8.5.1. Bias of the Estimate, 300
  - 8.5.2. Variance of the Estimate, 304
  - 8.5.3. Normalized rms Error, 305
  - 8.5.4. Estimates from Finite Fourier Transforms, 306
  - 8.5.5. Tests for Equivalence of Autospectra, 309
- 8.6. Record Length Requirements, 311

**9. Statistical Errors in Advanced Estimates**

316

- 9.1. Cross-Spectral Density Function Estimates, 316
  - 9.1.1. Variance Formulas, 319
  - 9.1.2. Covariance Formulas, 320
  - 9.1.3. Phase Angle Estimates, 325
- 9.2. Single-Input/Output Model Estimates, 326
  - 9.2.1. Bias in Frequency Response Function Estimates, 328
  - 9.2.2. Coherent Output Spectrum Estimates, 331
  - 9.2.3. Coherence Function Estimates, 333
  - 9.2.4. Gain Factor Estimates, 336
  - 9.2.5. Phase Factor Estimates, 338
- 9.3. Multiple-Input/Output Model Estimates, 341
  - 9.3.1. Multiple Coherence Function Estimates, 341
  - 9.3.2. Multiple Coherent Output Spectrum Estimates, 342
  - 9.3.3. Single Conditioned-Input/Output Models, 343

9.3.4.	Partial Coherence Function Estimates, 343	
9.3.5.	Partial Coherent Output Spectrum Estimates, 344	
9.3.6.	Gain Factor Estimates for Conditioned Models, 344	
9.3.7.	Phase Factor Estimates for Conditioned Models, 344	
9.3.8.	Modified Procedure Estimates. 345	
<b>10.</b>	<b>Data Acquisition and Processing</b>	<b>349</b>
10.1.	Data Acquisition, 349	
10.1.1.	Transducer and Signal Conditioning, 350	
10.1.2.	Data Transmission, 353	
10.1.3.	Calibration, 354	
10.1.4.	Dynamic Range, 357	
10.2.	Data Storage, 359	
10.2.1.	Magnetic Tape Recorders, 360	
10.2.2.	Modulation Procedures, 360	
10.2.3.	Digital Storage, 362	
10.3.	Data Conversion, 362	
10.3.1.	Analog-to-Digital Conversion, 362	
10.3.2.	Sampling Theorems for Random Records, 364	
10.3.3.	Sampling Rates and Aliasing Errors, 366	
10.3.4.	Quantization and Other Errors, 369	
10.4.	Data Qualification, 371	
10.4.1.	Data Classification, 372	
10.4.1.	Data Validation, 377	
10.4.3.	Data Editing, 382	
10.5.	Data Analysis Procedures, 385	
10.5.1.	Procedures for Analyzing Individual Records, 385	
10.5.2.	Procedures for Analyzing Multiple Records, 388	
<b>11.</b>	<b>Data Analysis</b>	<b>394</b>
11.1.	Data Preparation, 394	
11.1.1.	Data Standardization, 395	
11.1.2.	Trend Removal, 396	
11.1.3.	Digital Filtering, 398	
11.2.	Fourier Series and Fast Fourier Transforms, 401	
11.2.1.	Standard Fourier Series Procedures, 402	
11.2.2.	Fast Fourier Transforms, 403	
11.2.3.	Cooley-Tukey Procedure, 410	
11.2.4.	Procedures for Real-valued Records, 411	
11.2.5.	Further Related Formulas. 413	

11.2.6.	Other Algorithms, 415	
11.3.	Probability Density Functions, 416	
11.4.	Autocorrelation Functions 417	
11.4.1.	Autocorrelation Estimates via Direct Computations, 418	
11.4.2.	Autocorrelation Estimates via FFT Computations, 418	
11.5.	Autospectral Density Functions, 423	
11.5.1.	Autospectra Estimates by Ensemble Averaging, 423	
11.5.2.	Side-Lobe Leakage Suppression Procedures, 425	
11.5.3.	Recommended Computational Steps for Ensemble-Averaged Estimates, 432	
11.5.4.	Zoom Transform Procedures, 434	
11.5.5.	Autospectra Estimates by Frequency Averaging, 437	
11.5.6.	Other Spectral Analysis Procedures, 441	
11.6.	Joint Record Functions, 442	
11.6.1.	Joint Probability Density Functions, 443	
11.6.2.	Cross-Correlation Functions, 443	
11.6.3.	Cross-Spectral Density Functions, 445	
11.6.4.	Frequency Response Functions, 446	
11.6.5.	Unit Impulse Response (Weighting) Functions, 446	
11.6.6.	Ordinary Coherence Functions, 447	
11.7.	Multiple-Input/Output Functions, 447	
11.7.1.	Fourier Transforms and Spectral Functions, 447	
11.7.2.	Conditioned Spectral Density Functions, 449	
11.7.3.	Three-Input/Single-Output Models, 450	
11.7.4.	Functions in Modified Procedure, 453	
<b>12.</b>	<b>Nonstationary Data Analysis</b>	<b>457</b>
12.1.	Classes of Nonstationary Data, 457	
12.2.	Probability Structure of Nonstationary Data, 460	
12.2.1.	Higher-Order Probability Functions, 460	
12.2.2.	Time-Averaged Probability Functions, 462	
12.3.	Nonstationary Mean Values, 463	
12.3.1.	Independent Samples, 465	
12.3.2.	Correlated Samples, 466	
12.3.3.	Analysis Procedures for Single Records, 468	
12.4.	Nonstationary Mean Square Values, 470	
12.4.1.	Independent Samples, 470	
12.4.2.	Correlated Samples, 472	
12.4.3.	Analysis Procedures for Single Records, 473	

12.5.	Correlation Structure of Nonstationary Data, 477	
12.5.1.	Double-Time Correlation Functions, 477	
12.5.2.	Alternative Double-Time Correlation Functions, 478	
12.5.3.	Analysis Procedures for Single Records, 481	
12.6.	Spectral Structure of Nonstationary Data, 484	
12.6.1.	Double-Frequency Spectral Functions, 484	
12.6.2.	Alternative Double-Frequency Spectral Functions, 486	
12.6.3.	Frequency-Time Spectral Functions, 491	
12.6.4.	Analysis Procedures for Single Records, 499	
12.7.	Input/Output Relations for Nonstationary Data, 506	
12.7.1.	Nonstationary Input and Time-Varying System, 507	
12.7.2.	Results for Special Cases, 509	
12.7.3.	Frequency-Time Spectral Input/Output Relations, 510	
12.7.4.	Energy Spectral Input/Output Relations, 512	
<b>13.</b>	<b>The Hilbert Transform</b>	<b>518</b>
13.1.	Hilbert Transforms for General Records, 518	
13.1.1.	Computation of Hilbert Transforms, 521	
13.1.2.	Examples of Hilbert Transforms, 522	
13.1.3.	Properties of Hilbert Transforms, 522	
13.1.4.	Relation to Physically Realizable Systems, 526	
13.2.	Hilbert Transforms for Correlation Functions, 530	
13.2.1.	Correlation and Envelope Definitions, 530	
13.2.2.	Hilbert Transform Relations, 531	
13.2.3.	Analytic Signals for Correlation Functions, 532	
13.2.4.	Nondispersive Propagation Problems, 535	
13.2.5.	Dispersive Propagation Problems, 539	
13.3.	Envelope Detection Followed by Correlation, 543	
	<b>Bibliography</b>	<b>550</b>
	<b>Appendix A: Statistical Tables</b>	<b>554</b>
	<b>Appendix B: Definitions for Random Data Analysis</b>	<b>565</b>
	<b>List of Figures</b>	<b>577</b>
	<b>List of Tables</b>	<b>582</b>
	<b>List of Examples</b>	<b>583</b>
	<b>Index</b>	<b>586</b>