

Table of Contents

1	Introduction	1
2	Renormalized Perturbation Theory	3
2.1	Generating Functionals	4
2.1.1	The Green Functional	4
2.1.2	The Connected and the Vertex Functionals	6
2.1.3	Expansion in \hbar	7
2.1.4	Composite Fields	7
2.2	Renormalization	9
2.2.1	Ultraviolet Divergences	9
2.2.2	Ultraviolet Subtractions	11
2.2.3	Generalization to Higher Loop Graphs	15
2.2.4	General Results	16
2.2.5	Renormalizability and Nonrenormalizability	17
2.2.6	Physical and Nonphysical Renormalizations	18
2.2.7	The Role of Symmetry	19
2.3	Asymptotic Behaviour	20
3	Quantum Action Principle and Ward Identities	21
3.1	Ward Identities in the Tree Approximation	21
3.1.1	Linear Transformations	22
3.1.2	Nonlinear Rigid Invariance: BRS Transformations	23
3.2	The Quantum Action Principle	24
3.3	Basis of Insertions	26
3.4	A U(1) Scalar Model	27
3.4.1	Ward Identity	27
3.4.2	The Callan-Symanzik Equation	29
3.4.3	Renormalization Group Equation	31
3.5	A O(N) Scalar Model	32
3.5.1	Ward Identities	32
3.5.2	Solution of the Consistency Condition	33
3.5.3	The Callan-Symanzik Equation	34
3.6	Basis of Symmetric Insertions	35
3.7	Summary of the Quantum Action Principle	35

4 Yang-Mills Gauge Theories	37
4.1 Tree Approximation, Gauge Fixing and BRS Invariance	37
4.2 Renormalization: The Consistency Condition	40
4.3 BRS Cohomology: The Gauge Anomaly	45
4.3.1 Elimination of the External Field and Matter Field Dependence from the Anomaly	46
4.3.2 The Gauge Anomaly and the Descent Equations	47
4.4 Stability.	49
4.4.1 Invariant Counterterms	49
4.4.2 Physical and Nonphysical Parameters	50
4.4.3 The Callan-Symanzik Equation	51
4.5 Gauge Independence of the Physical Operators	52
4.6 Gauge Invariance Together with Rigid Invariance	54
4.6.1 BRS Transformations	54
4.6.2 Gauge Fixing	56
4.6.3 Slavnov-Taylor Identity and Ward Identities	56
5 BRS Cohomology and Descent Equations	59
5.1 The Descent Equations: Definitions and Generalities	59
5.1.1 Cohomology of δ Modulo d and Cohomology of δ	59
5.1.2 The General Strategy for Solving the Descent Equations	61
5.2 Local Cohomologies	62
5.2.1 Filtration	62
5.2.2 BRS Doublets	64
5.2.3 Restriction to a Subspace Constrained by a Rigid Symmetry	65
5.2.4 Poincare Lemma for d	66
5.3 General Solution of the Classical Descent Equations in Yang-Mills Theory	67
5.3.1 The Descent Equations of the Gauge Anomaly	67
5.3.2 General Results for Yang-Mills Theories in the Classical Approximation	71
5.3.3 Local Cohomology of s	71
5.3.4 Local Cohomology of s Modulo d . The Russian Formula	72
5.4 General Solution of the Classical Descent Equations in Topological Theory	73
5.5 Renormalized Descent Equations in Yang-Mills Theory	76
5.5.1 Renormalization	76
5.5.2 The Callan-Symanzik Equation for the Ladder of the Gauge Anomaly.	78
6 Nonrenormalization Theorems	81
6.1 The Antighost Equation	81
6.1.1 Renormalization of the Antighost Equation	83
6.2 Nonrenormalization Theorems	85
6.2.1 Nonrenormalization of the Ghost Field	85
6.2.2 Nonrenormalization of $\text{Tr } c^{(2p+1)}$	87
6.3 The Adler-Bardeen Theorem	89
6.3.1 The Callan-Symanzik Equation for the Anomaly	89
6.3.2 The Nonrenormalization Theorem of the Gauge Anomaly	91

7	Topological Field Theories	93
7.1	The Chern-Simons Model	93
7.2	The Vector Supersymmetry	95
7.2.1	The Off-Shell Algebra	97
7.3	Finiteness of the Chern-Simons Theory	99
7.3.1	Renormalization of the Slavnov-Taylor Identity in Three Dimensions	99
7.3.2	Renormalization of the Vector Supersymmetry	101
7.3.3	Ultraviolet Finiteness	102
7.3.4	Quantum Scale Invariance	103
7.4	BF Models	104
7.4.1	Gauge Fixing	105
7.4.2	Slavnov-Taylor Identity and Ghost Equations	108
7.4.3	Vector Supersymmetry and Antighost Equation	110
7.4.4	Renormalization	112
8	The Bosonic String	117
8.1	The Bosonic String in the Beltrami Parametrization	118
8.2	The Supersymmetry Algebra	122
8.3	The Diffeomorphism Anomaly and the Critical Dimension	124
8.4	The Algebra of the Energy-Momentum Tensor	126
	References	129