

FINITE MATHEMATICS

WITH APPLICATIONS IN THE SOCIAL
AND MANAGEMENT SCIENCES

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PREFACE

We have tried to steer a course through the Scylla and Charybdis of too much theory and not enough theory. We are sure we shall be criticized for both, but the result is our considered judgment of what is desirable. Too much theory for the nonprofessional mathematician tends to prevent the achievement of one important objective, viz., the ability to use these mathematical concepts in the modern world of business, government, and research. We do not subscribe to the unsupported hope that students equipped only with theory will know how to make the appropriate applications. On the other hand, too little theory fashions a student who may know how to apply techniques without a true comprehension of why these techniques do work. Without a basis in theory, the creative use of mathematics is almost impossible. To provide some experience in application and understanding, we have worked some of the applications in along with the theory, while others are provided in the form of straightforward use of techniques.

What we have tried to write is basically a mathematics book with applications in the social and management sciences. The early part of the book is relatively easy. As the student develops some maturity, the level of the work is raised. We have indicated from time to time certain portions of the book that may be omitted, either on first reading or by students whose level of maturity is not high. Accordingly, the book is very flexible and lends itself to a number of types of courses:

1. A liberal-arts course in finite mathematics
2. A course in a college of business administration either for freshmen or for more advanced students who need acquaintance with basic mathematical concepts

In addition, it is useful as a supplementary text in the following courses :

3. A course, at the senior or first-year graduate level, for students of the social and management sciences in preparation for quantitative analysis in their research courses and thesis work to come
4. A course in linear programming which requires a review of basic mathematics

Each instructor, naturally, will select from the material those chapters he finds most suitable for his particular students. For example, an instructor of students who fall in the third category above would want to go at a relatively fast pace and include Chaps. 12 and 13, which

apply the theory of vectors and matrices developed in Chaps. 7, 10, and 11 to linear programming. It is the availability of these chapters which also makes our text suitable to students falling in the fourth category.

On the other hand, an instructor offering a course to average students falling in either of the first two categories could omit these chapters, without disturbing the continuity of the text, and concentrate on the basic mathematics. In this event, the instructor will find ample opportunity to treat applications in these mathematical core chapters, since they contain examples, problems, and entire sections on application.

Again, within each chapter we have sought to provide flexibility by placing the application sections in such a position (usually at the end) that they can be omitted without affecting the continuity of the text. Examples of this are the sections dealing with logical models in production management (Chap. 1), Bayes theorem and binomial probabilities (Chap. 4), break-even analysis (Chap. 5), an introduction to linear programming (Chap. 6), the Leontief input/output model (Chap. 7), learning curves (Chap. 8), and capital theory and product failure (Chap. 9).

Above-average students at the first and second-year undergraduate level can, of course, be held responsible for some, or all, of the chapters excluded above.

There are certain distinctive features of the book that must be recorded :

1. Wherever possible, computational formulas are given in a form adaptable to machine programming.
2. Very often, a single problem recurs in more than one context. This enables the student to see the various aspects of a given problem.
3. The integrated and cohesive nature of mathematics is illustrated by the use of mathematical concepts developed in earlier chapters in the applications of later chapters. For example, the various probability definitions and theorems of Chap. 4 and the exponential function introduced in Chap. 8 are used in conjunction with the subject matter of Chap. 9 (sequences and progressions) in a product-life application presented at the end of Chap. 9, and the concepts of a random variable, probability distribution, and expected value introduced in Chap. 4 are applied in Chap. 10 to a decision-theory problem involving matrices.
4. Some nonfinite mathematics is introduced (e.g., Chaps. 5, 8, 9) which we thought important for further work. This material is organized so that it can be treated or not, at the discretion of the instructor.
5. We provide a self-contained development of linear programming.
6. Our examples and problems treat situations arising in economics, management, sociology, psychology, education, and political science. Most of these applications are from the fields of economics and business. Nonetheless, they are so constructed that they form paradigms for

structurally identical problems in any of the other sciences, both social and natural.

7. The chapters on linear programming have been put at the end of the book because of their complexity and the degree of maturity they demand for comprehension. We think that they are not above the level which we hope the student will have reached, but they do require rather sustained attention.

8. A rather complete study of linear algebra is included because of its importance.

9. We have introduced a fair amount of analytical geometry, where it occurs naturally.

10. We introduce, at an early stage, the concepts of sets and functions and use them as consistently as possible, departing from them only for pedagogical reasons.

11. Ample examples and problems are provided, in the belief that students can learn best by "doing." Those students who want to pursue further a given topic will find additional concepts, both theoretical and in the area of application, in these problem sections.

12. We have introduced topics with a view toward preparing the student for work in the calculus.

13. We have tried to conform as far as possible to the recommendations of the Committee on Undergraduate Programs in Mathematics for Biology, Management, and Social Sciences of the American Mathematical Association¹ and to the recommendations of M. D. Richards and R. Carzo in their Ford Foundation sponsored monograph "Mathematics in Collegiate Business Schools."² Most of the topics recommended by the above studies are discussed. Naturally, however, all we can do is lay the groundwork for further work. We therefore consider the book suitable for the modern student and presuppose only a good secondary school background in mathematics.

14. To enhance the teachability of the text, answers to odd numbered problems are provided in an answer section appearing at the end of the book, and an Instructor's Manual containing fully worked out solutions to most problems is available to instructors.

We wish to thank the administration of Boston College for their encouragement and assistance while this project was in progress.

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¹ Committee on the Undergraduate Program in Mathematics, "Tentative Recommendations for the Undergraduate Mathematics Program of the Students in the Biological, Management, and Social Sciences," Mathematical Association of America, 1964.

² M. D. Richards and R. Carzo, "Mathematics in Collegiate Business Schools," South-Western Publishing Co., Cincinnati, 1963.

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