

Contents

1 USING STATISTICS	1
2 DESCRIBING THE DATA.	3
2.1 Types of Data	3
2.2 Bar Charts and Histograms	4
2.3 Averages	6
2.3.1 The arithmetic mean	6
★ 2.3.2 Alternatives to the arithmetic mean	7
2.4 Measuring the Spread	8
2.4.1 The variance	8
2.4.2 The standard deviation	9
★ 2.4.3 Different definitions of the standard deviation	10
★ 2.4.4 Alternative measures of the spread	12
★ 2.5 Higher Powers of x	13
★ 2.5.1 Skew	13
★ 2.5.2 Higher powers	14
2.6 More Than One Variable	14
2.6.1 Covariance	15

2.6.2 Correlation	15
★ 2.6.3 More than two variables	17
2.7 Problems	18
3 THEORETICAL DISTRIBUTIONS	20
3.1 General Properties of Distributions	21
3.1.1 A simple distribution	21
3.1.2 The law of large numbers	21
3.1.3 Expectation values	22
3.1.4 Probability density distributions	23
3.2 The Binomial Distribution	24
3.2.1 The binomial probability distribution formula	25
★ 3.2.2 Proof of properties of the binomial distribution	27
3.3 The Poisson Distribution	28
3.3.1 The Poisson probability formula	29
★ 3.3.2 Proof of properties of the Poisson distribution	32
★ 3.3.3 Two Poisson distributions	33
3.4 The Gaussian Distribution	34
3.4.1 The Gaussian probability distribution function.	34
★ 3.4.2 Proof of properties of the Gaussian.	36
3.4.3 Definite integrals.	36
3.4.4 Indefinite integrals	37
3.4.5 Gaussian as limit of the Poisson and binomial.	40
★ 3.4.6 The many-dimensional Gaussian	41
★ 3.4.7 The binormal distribution	42
★ 3.5 Other Distributions.	44
★ 3.5.1 The uniform distribution	45
★ 3.5.2 The Weibull distribution	45
★ 3.5.3 The Breit–Wigner or Cauchy distribution	46
3.6 Problems	46
4 ERRORS.	48
4.1 Why Errors are Gaussian	49
4.1.1 The central limit theorem	50
4.2 Working with Errors	51
4.2.1 Repeated measurements	51
4.2.2 Averaging weighted measurements	53
4.2.3 A note of caution	54
4.3 Combination of Errors	55
4.3.1 One variable	55
4.3.2 A function of two or more variables	56
4.3.3 Percentage errors	58

★ 4.3.4 Several functions of several variables	58
4.4 Systematic Errors	61
4.4.1 Finding, eliminating, and evaluating them	62
★ 4.4.2 Living with systematic errors	65
4.5 Problems	67
5 ESTIMATION	68
5.1 Properties of Estimators	68
5.1.1 Consistency, bias, and efficiency.	69
★ 5.1.2 The likelihood function	71
★ 5.1.3 Proof of the minimum variance bound	74
5.2 Some Basic Estimators	75
5.2.1 Estimating the mean.	76
5.2.2 Estimating the variance	76
5.2.3 Estimating σ	78
★ 5.2.4 The correlation coefficient	80
★ 5.3 Maximum Likelihood	81
★ 5.3.1 ML: consistency, bias, and invariance	84
★ 5.3.2 Maximum likelihood at large N	85
★ 5.3.3 Errors on the ML estimators	86
★ 5.3.4 Several variables	88
★ 5.3.5 Notes on maximum likelihood	89
★ 5.4 Extended Maximum Likelihood	90
★ 5.5 The Method of Moments	92
★ 5.6 Maximum Likelihood and Least Squares	93
★ 5.7 Stratified Sampling—Beating \sqrt{N}	93
5.8 Problems	95
6 LEAST SQUARES	97
6.1 Outline of the Method	97
6.1.1 Fitting $y = mx$ – simple proportion	98
6.2 The Straight Line Fit	99
6.2.1 The slope and intercept for a straight line	100
★ 6.2.2 Derivation of the errors for a straight line	101
★ 6.2.3 Weighted straight line fit	102
★ 6.2.4 Extrapolation	103
★ 6.2.5 Systematic errors and a straight line fit	103
★ 6.2.6 Regression	104
6.3 Fitting Binned Data	105
6.4 The χ^2 Distribution.	106
★ 6.4.1 Proof of the χ^2 distribution	108
★ 6.5 Errors on x and y	109

★ 6.6 Linear Least Squares and Matrices	111
★ 6.6.1 Straight line fit using matrices	113
★ 6.6.2 Higher polynomials	114
★ 6.7 Non-linear Least Squares	115
6.8 Problems	116
★ 7 PROBABILITY AND CONFIDENCE	118
★ 7.1 What is Probability?	119
★ 7.1.1 Mathematical probability	119
★ 7.1.2 Empirical—the limit of a frequency	119
★ 7.1.3 Objective—propensity	120
★ 7.1.4 Subjective probability	121
★ 7.1.5 Bayesian statistics	123
★ 7.1.6 Conclusions on probability	124
★ 7.2 Confidence Levels	125
★ 7.2.1 Confidence levels in descriptive statistics	125
★ 7.2.2 Confidence intervals in estimation	127
★ 7.2.3 Confidence levels from Gaussians	129
★ 7.2.4 Measurement of a constrained quantity	130
★ 7.2.5 Binomial confidence intervals	132
★ 7.2.6 Poisson confidence intervals	133
★ 7.2.7 Several variables—confidence regions	134
★ 7.3 Student's <i>t</i> Distribution	134
★ 7.3.1 Proof of the formula for Student's <i>t</i>	138
★ 7.4 Problems	139
8 TAKING DECISIONS	141
8.1 Hypothesis Testing	142
8.1.1 Hypotheses	142
8.1.2 Type I and type II errors	142
8.1.3 Significance	143
8.1.4 Power	143
★ 8.1.5 The Neyman Pearson test	144
8.2 Interpreting Experiments	145
8.2.1 The null hypothesis	146
★ 8.2.2 Binomial probabilities	147
★ 8.2.3 Is there a signal??—Poisson statistics	148
8.3 Goodness of Fit	149
8.3.1 The χ^2 test	150
★ 8.3.2 The run test	153
★ 8.3.3 The Kolmogorov test	155
8.4 The Two-sample Problem	156

8.4.1 Two Gaussian samples with known σ	156
★ 8.4.2 Two Gaussian samples with unknown σ	157
★ 8.4.3 Matched and correlated samples	159
★ 8.4.4 The <i>F</i> distribution	160
★ 8.4.5 The general case	161
★ 8.5 Analysis Methods for Several Samples	164
★ 8.5.1 The analysis of variance (basic method)	164
★ 8.5.2 Multiway analysis of variance	166
★ 8.5.3 Contingency tables	169
8.6 Problems	170
★ 9 RANKING METHODS	172
★ 9.1 Non-parametric Methods	173
★ 9.1.1 The sign test for the median	173
★ 9.2 Two Ranked Samples	174
★ 9.2.1 The Mann-Whitney test	174
★ 9.2.2 Matched pairs	175
★ 9.2.3 Wilcoxon's matched pairs signed rank test	176
★ 9.3 Measures of Agreement	177
★ 9.3.1 Spearman's correlation coefficient	177
★ 9.3.2 Concordance	178
★ 9.4 Problems	179
10 NOTES FOR NUMBER CRUNCHERS	180
10.1 Significance	180
10.1.1 Subtraction	181
10.1.2 Computing the standard deviation	181
10.1.3 Addition	182
10.1.4 Quadratic equations	183
10.2 Other Do's and Don'ts	183
10.2.1 Matrix inversion	183
10.2.2 Fitting curves	184
10.3 Random Number Generation	184
10.4 Style	185
Bibliography	187
Appendix 1 Answers to Problems	190
Appendix 2 Proof of the Central Limit Theorem	196
Index	199