

# Contents of Volume 1

<b>Contents of Volume 2</b>	<b>x</b>
<b>Preface</b>	<b>xiii</b>
<b>Organization of the Book</b>	<b>xvii</b>
<b>Acknowledgments</b>	<b>xxi</b>
<b>Introduction</b>	<b>xxiii</b>

## **PART ONE: THE RAW MATERIALS OF MATHEMATICS 1**

<b>Chapter 1 Sets</b>	<b>3</b>
1.1 Operations with Sets	6
1.2 Relations in Sets	13
1.2a. Equivalence relations	15
1.2b. Order relations	20
<b>Chapter 2 Maps</b>	<b>26</b>
2.1 Composite Functions and Inverses	32
2.2 Equivalence Relations and Maps	38
2.3 Ordered Sets and Maps	42
2.4 Cardinal Numbers	43
2.5 Sequences and Families	47

**PART TWO: THE BASIC STRUCTURES OF MATHEMATICS 53***IIA: Algebraic Structures 55***Chapter 3 Algebraic Composition Laws and Systems 57**  
3.1 Morphisms of Algebraic Systems 62**Chapter 4 Survey of Special Algebraic Systems 68**

- 4.1 Groups 70
  - 4.1a. Transformation groups;  $G$ -spaces; orbits 79
  - 4.1b. Conjugate classes; cosets 87
  - 4.1c. Normal subgroups; quotient groups; isomorphism theorems 91
- 4.2 Rings and Fields 102
  - 4.2a. Ideals; quotient rings; isomorphism theorems 115
- 4.3 Linear Spaces 120
  - 4.3a. Linear independence, bases and dimension 128
  - 4.3b. Morphisms (linear transformations); quotient spaces 137
- 4.4 Linear Algebras 149
  - 4.4a. Morphisms of algebras; quotient algebras 159
- 4.5 Nonassociative Algebras 167
  - 4.5a. Lie algebras 168
  - 4.5b. Some other nonassociative algebras 182

*IIB: Topological Structures 185***Chapter 5 Topological Spaces 187**

- 5.1 Examples; Metric Spaces 188
- 5.2 General Structure of Topological Spaces 198
- 5.3 Neighborhoods; Special Points; Closed Sets 203
  - 5.3a. Interior, closure, boundary 208
- 5.4 Convergence 211
- 5.5 Continuity 216
- 5.6 Homeomorphism and Isometry 220
  - 5.6a. Quotient topology; homeomorphism theorem 229

**Chapter 6 Topological Spaces with Special Properties 236**

- 6.1 Connected Spaces 236
  - 6.1a. Path connectivity; homotopy 242
- 6.2 Separable Spaces 250

- 6.3 Compact Spaces 254
  - 6.3a. Compactification 266
- 6.4 Complete Metric Spaces 270
  - 6.4a. Completion 275
  - 6.4b. Contraction mappings 280

*IIC: Measure Structures 291***Chapter 7 Measure Spaces 293**

- 7.1 Measurable Spaces 294
- 7.2 Measure and Measure Spaces 305
  - 7.2a. General properties of measures 310
  - 7.2b. Lebesgue measure 314
  - 7.2c. Lebesgue–Stieltjes measures 320
  - 7.2d. Signed and complex measure 325

**Chapter 8 Theory of Integration 328**

- 8.1 Measurable Functions 329
- 8.2 Definition of the Integral 338
- 8.3 General Properties of the Integral 353
- 8.4 Comments on Lebesgue and Lebesgue–Stieltjes Integrals 365
- 8.5 The Radon–Nikodym Theorem 371

**APPENDICES xxvii****Appendix I Some Inequalities xxix****Appendix III Annotated Reading List xxx****Appendix IV Frequently Used Symbols xxxvi****Index xliii**

# Contents of Volume 2

<b>PART THREE: COMBINATION OF SYSTEMS: FUNCTIONAL ANALYSIS</b>	<b>379</b>
<i>IIIA: Topological Linear Spaces</i>	<i>379</i>
<b>Chapter 9 Banach Spaces</b>	<b>381</b>
9.1 General Concepts Concerning Topological Linear Spaces	383
9.2 Normed Linear Spaces	394
9.3 Basic Facts About Banach Spaces	407
<b>Chapter 10 Hilbert Spaces</b>	<b>417</b>
10.1 Inner Product Spaces	417
10.2 Orthonormal Sets	425
10.3 Basic Facts About Hilbert Spaces	435
10.4 Orthonormal Expansions in Hilbert Spaces	440
10.5 Orthogonal Complements and Direct Sums	450
10.6 Weak Convergence of Vectors	459
<i>IIIB: Mappings of Topological Linear Spaces</i>	<i>463</i>
<b>Chapter 11 Linear Functionals</b>	<b>465</b>
11.1 Continuous Linear Transformations	466
11.2 Basic Properties of Continuous Linear Functionals	475
11.3 Dual Spaces and the Riesz Representation Theorem	480

<b>Chapter 12 Linear Operators</b>	<b>490</b>
12.1 Composites and Inverses of Linear Operators	491
12.2 Bounded Linear Operators	495
12.2a. The Banach algebra of bounded linear operators	501
12.2b. Extension of bounded linear operators	507
12.2c. Uniform, strong, and weak convergence of operators	511
12.2d. Closed operators and the closure of an operator	514
12.3 Hilbert Space Operators with Special Properties	518
12.3a. The adjoint of an operator	518
12.3b. Hermitean, selfadjoint, and normal operators	531
12.3c. Isometric and unitary operators	551
12.3d. Projection operators	565
<b>Chapter 13 Spectral Theory</b>	<b>579</b>
13.1 Resolvent and Spectrum	580
13.2 The Spectra of Normal, Hermitean, Selfadjoint, and Unitary Operators	598
13.3 The Spectra of Compact Operators	610
13.4 Spectral Representations	625
13.4a. Compact selfadjoint operators	625
13.4b. Selfadjoint operators and their functions	633
13.4c. Unitary operators and related topics	656

**APPENDICES** **xxiii**

<b>Appendix I Some Inequalities</b>	<b>xxv</b>
<b>Appendix II Generalized Functions and Distributions</b>	<b>xxvi</b>
<b>Appendix III Annotated Reading List</b>	<b>xliv</b>
<b>Appendix IV Frequently Used Symbols</b>	<b>li</b>
<b>Index</b>	<b>lvii</b>