

Contents

<i>Foreword</i>	v
<i>Acknowledgments</i>	xxv
1. General introduction	1
2. General background	7
2.1 Introduction	7
2.2 The two interacting systems: atom and field	9
2.2.1 External and internal atomic variables	9
2.2.2 Classical versus quantum treatments of atomic variables	10
2.2.3 Classical description of field variables	10
2.2.4 Quantum description of field variables	11
2.2.5 Atom-field interaction Hamiltonian in the long wavelength approximation	12
2.2.6 Elementary interaction processes	14
2.3 Basic conservation laws	14
2.3.1 Conservation of the total linear momentum	14
2.3.2 Conservation of the total angular momentum	17
2.4 Two-level atom interacting with a coherent monochromatic field. The Rabi oscillation	20
2.4.1 A simple case: magnetic resonance of a spin 1/2	20
2.4.2 Extension to any two-level atomic system	23
2.4.3 Perturbative limit	25
2.4.4 Two physical pictures for Ramsey fringes	27
2.5 Two-level atom interacting with a broadband field. Absorption and emission rates	29
2.5.1 Absorption rate deduced from a semiclassical treatment of the field	29
2.5.2 Physical discussion. Relaxation time and correlation time	31

2.5.3	Sketch of a quantum treatment of the absorption process	31
2.5.4	Extension to spontaneous emission	32
2.6	Two-level atom interacting with a coherent monochromatic field in the presence of damping	33
	Light: a source of information on atoms	35
3.	Optical methods	41
3.1	Introduction	41
3.2	Double resonance	43
3.2.1	Principle of the method	43
3.2.2	Predicted shape for the double resonance curve	44
3.2.3	Experimental results	45
3.2.4	Interpretation of the Majorana reversal	45
3.3	Optical pumping [Kastler (1950)]	46
3.3.1	Principle of the method for a $J_g = 1/2 \rightarrow J_e = 1/2$ transition	47
3.3.2	Angular momentum balance	48
3.3.3	Double role of light	48
3.4	First experiments on optical pumping	49
3.5	How can optical pumping polarize atomic nuclei?	49
3.5.1	Using hyperfine coupling with polarized electronic spins	49
3.5.2	First example: optical pumping experiments with mercury-199 atoms	52
3.5.3	Second example: combining optical pumping with metastability exchange collisions for helium-3	52
3.5.4	A new application: magnetic resonance imaging of the lung cavities	54
3.6	Brief survey of the main applications of optical methods	55
3.7	Concluding remarks	58
4.	Linear superpositions of internal atomic states	61
4.1	Introduction	61
4.2	First experimental evidence of the importance of atomic coherences	62
4.3	Zeeman coherences in excited states	64
4.3.1	How to prepare Zeeman coherences in excited states e ?	64
4.3.2	Physical interpretation	64
4.3.3	How to detect Zeeman coherences in e ?	66
4.3.4	Equation of motion of Zeeman coherences in e	66
4.3.5	Level crossing resonances in the excited state e	67

4.3.6	Pulsed excitation. Quantum beats	69
4.3.7	Excitation with modulated light	70
4.3.8	Modulation of the fluorescence light in a double resonance experiment. Light beats	70
4.4	Zeeman coherences in atomic ground states	71
4.4.1	Hanle effect in atomic ground states	71
4.4.2	Detection of the magnetic resonance in the ground state by the modulation of the absorbed light	74
4.5	Transfer of coherences	74
4.6	Dark resonances. Coherent population trapping	79
4.6.1	Discovery of dark resonances	79
4.6.2	First theoretical treatment of dark resonances	80
4.6.3	Interpretation of the Raman resonance condition	81
4.6.4	A few applications of dark resonances	82
4.7	Conclusion	84
5.	Resonance fluorescence	87
5.1	Introduction	87
5.2	Low intensity limit. Perturbative approach	88
5.2.1	Lowest order process	88
5.2.2	Resonant scattering amplitude	89
5.2.3	Scattering of a light wave packet	91
5.2.4	First higher order processes	92
5.3	Optical Bloch equations	93
5.4	The dressed atom approach	96
5.4.1	The interacting systems	96
5.4.2	Uncoupled states of the atom-laser system	97
5.4.3	Effect of the coupling. Dressed states	98
5.4.4	Two different situations	99
5.4.5	Radiative cascade in the basis of uncoupled states	101
5.4.6	A new description of quantum dissipative processes	104
5.5	Photon correlations. The quantum jump approach	105
5.5.1	The waiting time distribution	105
5.5.2	From the waiting time distribution to the second order correlation function	106
5.5.3	Photon antibunching	106
5.6	Fluorescence triplet at high laser intensities	108
5.6.1	Limit of large Rabi frequency	108
5.6.2	Mollow fluorescence triplet	108
5.6.3	Widths and weights of the components of the Mollow triplet	110

5.6.4	Time correlations between the photons emitted in the two sidebands of the fluorescence triplet	111
5.7	Conclusion	112
6.	Advances in high resolution spectroscopy	115
6.1	Introduction	115
6.2	Saturated absorption	117
6.2.1	Principle of the method	117
6.2.2	Crossover resonances	118
6.2.3	Recoil doublet	120
6.3	Two-photon Doppler-free spectroscopy	121
6.3.1	Principle of the method	121
6.3.2	Examples of results	122
6.3.3	Comparison between saturated absorption and two-photon spectroscopy	124
6.4	Recoil suppressed by confinement: the Lamb-Dicke effect	124
6.4.1	Intensities of the vibrational lines	125
6.4.2	Influence of the localization of the ion	127
6.4.3	Case of a harmonic potential	127
6.4.4	Historical perspective	128
6.5	The shelving method	129
6.5.1	Single ion spectroscopy	130
6.5.2	Intermittent fluorescence	131
6.5.3	Properties of the detected signal	132
6.5.4	Observation of quantum jumps	134
6.6	Quantum logic spectroscopy	135
6.7	Frequency measurement with frequency combs	137
6.8	Conclusion	139

Atom-photon interactions: a source of perturbations for atoms which can be useful 141

7.	Perturbations due to a quasi resonant optical excitation	145
7.1	Introduction	145
7.2	Light shift, light broadening and Rabi oscillation	147
7.2.1	Effective Hamiltonian	147
7.2.2	Weak coupling limit. Light shift and light broadening	148
7.2.3	High coupling limit. Rabi oscillation	149
7.2.4	Absorption rate versus Rabi oscillation	151
7.2.5	Semiclassical interpretation in the weak coupling limit	151
7.2.6	Generalization to a non-resonant excitation	152

7.2.7	Case of a degenerate ground state	154
7.3	Perturbation of the field. Dispersion and absorption	155
7.3.1	Atom in a cavity	155
7.3.2	Frequency shift of the field due to the atom	156
7.3.3	Damping of the field	157
7.4	Experimental observation of light shifts	158
7.4.1	Principle of the experiment	158
7.4.2	Examples of results	159
7.5	Using light shifts for manipulating atoms	161
7.5.1	Laser traps	161
7.5.2	Atomic mirrors	162
7.5.3	Blue detuned traps: a few examples	163
7.5.4	Optical lattices	164
7.5.5	Internal state dependent optical lattices	165
7.5.6	Coherent transport	167
7.6	Using light shifts for manipulating fields	167
7.6.1	Linear superposition of two field states with different phases	168
7.6.2	Non-destructive detection of photons	168
7.7	Conclusion	169
8.	Perturbations due to a high frequency excitation	171
8.1	Introduction	171
8.2	Spin 1/2 coupled to a high frequency RF field	173
8.2.1	Hamiltonian	174
8.2.2	Perturbative treatment of the coupling	174
8.2.3	Stimulated corrections	176
8.2.4	Radiative corrections	177
8.3	Weakly bound electron coupled to a high frequency field	178
8.3.1	Effective Hamiltonian describing the modifications of the dynamical properties of the electron	178
8.3.2	Stimulated effects	180
8.3.3	Spontaneous effects. Vacuum fluctuations and radiation reaction	182
8.4	New insights into radiative corrections	183
8.4.1	Examples of spontaneous corrections	183
8.4.2	Interpretation of the Lamb shift	185
8.4.3	Interpretation of the spin anomaly $g - 2$	186
8.5	Conclusion	188

Atom-photon interactions: a simple system for studying higher order effects 191

9. Multiphoton processes between discrete states	195
9.1 Introduction	195
9.2 Radiofrequency multiphoton processes	196
9.2.1 Multiphoton RF transitions between two Zeeman sublevels m_F and $m_F + 2$	196
9.2.2 Experimental observation on sodium atoms	198
9.2.3 Multiphoton resonances between two Zeeman sublevels m_F and $m_F + 1$	199
9.3 Radiative shift and radiative broadening of multiphoton resonances	202
9.3.1 Energy levels of the atom+RF photons system. Transition amplitude	202
9.3.2 Pure single photon resonance. Simple anticrossing	204
9.3.3 Higher order anticrossing for a p -photon resonance ($p > 1$)	205
9.3.4 Application to the case of a spin 1/2 coupled to a σ -polarized RF field	206
9.4 Optical multiphoton processes between discrete states	211
9.4.1 Introduction	211
9.4.2 Radiative shift of Doppler-free two-photon resonances	211
9.4.3 Stimulated Raman processes	212
9.4.4 Phase matching condition. Application to degenerate four-wave mixing	217
9.5 Conclusion	219
10. Photoionization of atoms in intense laser fields	221
10.1 Introduction	221
10.2 Multiphoton ionization	223
10.2.1 Parameters influencing the multiphoton ionization rate	223
10.2.2 Quantum interference effects in multiphoton ionization	225
10.2.3 Asymmetric line profiles in resonant multiphoton ionization	226
10.3 Above threshold ionization (ATI)	227
10.3.1 Multiphoton transitions between states of the continuum	227
10.3.2 Consequences of the oscillatory motion of the electron in the laser field	228
10.3.3 Evidence for non-perturbative effects	229
10.4 Harmonic generation	231
10.4.1 Physical interpretation	231

10.4.2 High order harmonic generation (HHG). Evidence for non-perturbative effects	232
10.5 Tunnel ionization and recollision	233
10.5.1 The breakdown of perturbation theory	233
10.5.2 Keldysh parameter	234
10.5.3 Two-step quantum-classical model	235
10.5.4 Recollision	237
10.5.5 Full quantum treatments	239
10.6 Conclusion	239
Atom-photon interactions: a tool for controlling and manipulating atomic motion	241
11. Radiative forces exerted on a two-level atom at rest	247
11.1 Introduction	247
11.1.1 Order of magnitude of the force	247
11.1.2 Characteristic times	248
11.1.3 Validity of the concept of a mean force at a given point	249
11.2 Calculation of the mean radiative force	250
11.2.1 Principle of the calculation	250
11.2.2 Hamiltonian and the rotating wave approximation	251
11.2.3 Heisenberg equations for the external variables. Force operator	252
11.2.4 Approximations. Mean radiative force	253
11.2.5 The two types of mean radiative forces: dissipative and reactive	253
11.3 Dissipative force	256
11.3.1 Theoretical results	256
11.3.2 Physical interpretation	257
11.3.3 Application to the deflection and to the slowing down of an atomic beam	258
11.3.4 Fluctuations	260
11.4 Reactive force	261
11.4.1 Theoretical results	262
11.4.2 Physical interpretation	262
11.4.3 Dressed atom interpretation	263
11.5 Conclusion	266
12. Laser cooling of two-level atoms	269
12.1 Introduction	269
12.2 Doppler-induced friction force	271

12.2.1	Doppler effect in a red detuned laser plane wave	271
12.2.2	Low velocity behavior of the force	272
12.2.3	Idea of Doppler cooling for trapped ions	273
12.2.4	Idea of Doppler cooling for neutral atoms	273
12.3	Two-level atom moving in a weak standing wave. Doppler cooling	275
12.3.1	Perturbative approach for calculating the force	275
12.3.2	Friction coefficient for a red-detuned weak standing wave	276
12.3.3	Momentum-energy balance. Entropy balance	276
12.3.4	Limits of Doppler cooling. Lowest temperature	277
12.3.5	Consistency of the various approximations	279
12.3.6	Spatial diffusion. Optical molasses	279
12.4	Beyond the perturbative approach	280
12.4.1	Optical Bloch equations for a moving atom	280
12.4.2	Time lag of internal variables	281
12.4.3	Low velocity limit ($k_L v \ll \Gamma$)	282
12.4.4	Higher velocities	282
12.5	Dressed atom approach to atomic motion in an intense standing wave. Blue cooling	284
12.5.1	Energy and radiative widths of the dressed states	284
12.5.2	Friction mechanism	285
12.5.3	High intensity Sisyphus cooling	286
12.5.4	Experimental results	288
12.6	Conclusion	289
13	Sub-Doppler cooling. Sub-recoil cooling	291
13.1	Introduction	291
13.2	Sub-Doppler cooling	293
13.2.1	The basic ingredients of sub-Doppler cooling	293
13.2.2	Laser configuration and atomic transition	294
13.2.3	Light shifts and optical pumping for an atom at rest	294
13.2.4	Low intensity Sisyphus cooling for a moving atom	296
13.2.5	Characteristics of the friction force. Qualitative discussion	298
13.2.6	Quantum limits of sub-Doppler cooling	300
13.3	Sub-recoil cooling	302
13.3.1	Physical mechanism	302
13.3.2	Velocity selective coherent population trapping (VSCPT)	304
13.3.3	Sub-recoil Raman cooling	308
13.3.4	Quantitative predictions for sub-recoil cooling	310
13.4	Resolved sideband cooling of trapped ions	312
13.5	Conclusion	314

14	Trapping of particles	317
14.1	Introduction	317
14.2	Trapping of charged particles	318
14.2.1	The Earnshaw theorem	318
14.2.2	The Penning trap	319
14.2.3	The Paul trap	321
14.2.4	Cooling of the trapped ions	323
14.2.5	High precision measurements performed with ultracold trapped ions	324
14.3	Magnetic traps	325
14.3.1	Introduction	325
14.3.2	Quadrupole trap and Majorana losses	326
14.3.3	Ioffe-Pritchard trap	327
14.3.4	Time-averaged orbiting potential (TOP)	329
14.3.5	Loading neutral atoms in a magnetic trap	330
14.4	Electric dipole traps	330
14.4.1	Induced dipole moment	330
14.4.2	Application of dipole forces to trapping	332
14.4.3	Optical lattices	335
14.5	Artificial orbital magnetism for neutral atoms	338
14.5.1	Introduction	338
14.5.2	Rotating a harmonically trapped quantum gas	338
14.5.3	Artificial gauge potential from adiabatic evolution	339
14.6	Magneto-optical trap (MOT)	341
14.7	Conclusion	344

Ultracold interactions and their control 347

15	Two-body interactions at low temperatures	351
15.1	Introduction	351
15.2	Quantum scattering: a brief reminder	352
15.2.1	Scattering amplitude	353
15.2.2	Scattering cross section	355
15.2.3	Partial wave expansion	355
15.3	Scattering length	358
15.3.1	Low-energy limit	358
15.3.2	Scattering amplitude and scattering length	360
15.3.3	Square potential and resonances	361
15.3.4	Effective interactions and the sign of the scattering length	363
15.4	Pseudo-potential	365
15.4.1	Motivation for introducing this pseudo-potential	365

15.4.2	Localized pseudo-potential giving the correct scattering length	365
15.4.3	Scattering amplitude. Validity of the Born approximation	367
15.4.4	Bound state of the pseudo-potential for a positive scattering length	368
15.5	Delta potential truncated in momentum space	369
15.5.1	Expression of the potential	369
15.5.2	Determination of the new coupling constant	369
15.5.3	Comparison with the pseudo-potential	370
15.6	Forward scattering	371
15.6.1	Gaussian incident wave and scattered wave	371
15.6.2	Interference of the incident and scattered waves in the far-field zone	373
15.6.3	Phase shift of the incident wave and mean field energy	375
15.7	Conclusion	377
16.	Controlling atom-atom interactions	379
16.1	Introduction	379
16.2	Collision channels	380
16.2.1	Microscopic interactions	380
16.2.2	Quantum numbers of the initial collision state. Collision channels	382
16.2.3	Coupled channel equations	382
16.2.4	Two-channel model	383
16.3	Qualitative discussion. Analogy between Feshbach resonances and resonant light scattering	384
16.4	Scattering states of the two-channel Hamiltonian	386
16.4.1	Calculation of the dressed scattering states	386
16.4.2	Existence of a resonance in the scattering amplitude	388
16.4.3	Asymptotic behavior of the dressed scattering states	389
16.4.4	Scattering length. Feshbach resonance	391
16.5	Bound states of the two-channel Hamiltonian	393
16.5.1	Calculation of the energy of the bound state	393
16.5.2	Wave function of the bound state	396
16.5.3	Halo states	397
16.6	Producing ultracold molecules	399
16.6.1	Magnetic tuning of a Feshbach resonance	399
16.6.2	Photoassociation of ultracold atoms	400
16.7	Conclusion	402

Exploring quantum interferences with few atoms and photons		405
17.	Interference of atomic de Broglie waves	409
17.1	Introduction	409
17.2	De Broglie waves versus optical waves	410
17.2.1	Dispersion relations. Position and momentum distributions	410
17.2.2	Spatial coherences. Coherence length	411
17.2.3	Fragility of spatial coherences	413
17.3	Young's two-slit interferences with atoms	414
17.3.1	Important parameters of Young's double-slit interferometer	414
17.3.2	Young's double-slit interferences with supersonic beams	415
17.3.3	Young's double-slit interferences with cold atoms	416
17.3.4	Can one determine which slit the atom passes through?	417
17.4	Diffraction of atoms by material structures	418
17.5	Diffraction by laser standing waves	420
17.5.1	New features compared to the diffraction by material gratings	420
17.5.2	Light-atom momentum exchange	422
17.5.3	Raman-Nath regime	423
17.5.4	Bragg regime	424
17.6	Bloch oscillations	427
17.6.1	Review on the quantum treatment of a particle in a periodic potential	427
17.6.2	Implementation with cold atoms	428
17.6.3	Physical interpretations	430
17.7	Diffraction of atomic de Broglie waves by time-dependent structures	431
17.7.1	Phase modulation of atomic de Broglie waves	432
17.7.2	Atomic wave diffraction and interference using temporal slits	433
17.8	Conclusion	433
18.	Ramsey fringes revisited and atomic interferometry	435
18.1	Introduction	435
18.2	Microwave atomic clocks with cold atoms	437
18.2.1	Principle of an atomic clock	437
18.2.2	Atomic fountains	437
18.2.3	Performances of atomic fountains	438
18.2.4	Cold atoms clocks in space	441
18.2.5	Tests of general relativity	441

18.3	Extension of Ramsey fringes to the optical domain	442
18.3.1	Equivalence of the crossing of a laser beam with a coherent beam splitter	442
18.3.2	Spatial separation of the two final wave packets. Quenching of the interference	443
18.3.3	How to restore the interference signal?	444
18.3.4	Other possible schemes	448
18.4	Calculation of the phase difference between the two arms of an atomic interferometer	449
18.4.1	Quantum propagator and Feynman path integral	450
18.4.2	Simple case of quadratic Lagrangians	451
18.4.3	Phase shift in the absence of external potentials and inertial fields	452
18.4.4	Phase shift due to external potentials and inertial fields in the perturbative limit	453
18.5	Applications of atomic interferometry	454
18.5.1	Measurement of gravitational fields. Gravimeters	454
18.5.2	Measurement of rotational inertial fields	457
18.5.3	Measurement of h/M and α	459
18.6	New perspectives opened by optical clocks	461
19.	Quantum correlations. Entangled states	463
19.1	Introduction	463
19.2	Interference effects in double counting rates	464
19.2.1	Photodetection signals	464
19.2.2	Two-mode model for the light field	465
19.2.3	What are the "objects" which interfere in w_{II} ?	466
19.2.4	Establishment of correlations between the two modes	467
19.3	Entangled states	469
19.3.1	Definition	469
19.3.2	Schmidt decomposition of an entangled state	469
19.3.3	Information content of an entangled state	471
19.4	Preparing entangled states	472
19.4.1	Entanglement between one atom and one field mode	472
19.4.2	Entanglement between two atoms	473
19.4.3	Entanglement between two separate cavity fields	475
19.4.4	Entanglement between two photons	475
19.5	Entanglement and interference	477
19.6	Entanglement and non-separability	479
19.6.1	The Einstein-Podolsky-Rosen (EPR) argument [Einstein <i>et al.</i> (1935)]	479
19.6.2	Bell's inequalities	480

19.6.3	Experimental results and conclusion	481
19.7	Entanglement and which-path information	485
19.8	Entanglement and the measurement process	486
19.8.1	Von Neumann model of an ideal measurement process	486
19.8.2	Difficulty associated with macroscopic coherences	487
19.8.3	A possible solution: coupling of \mathcal{M} with the environment	487
19.8.4	Simple example of pointer states	488
19.8.5	The infinite chain of Von Neumann	489
19.9	Conclusion	490
Degenerate quantum gases		491
20.	Emergence of quantum effects in a gas	497
20.1	Introduction	497
20.2	Quantum effects in collisions	499
20.2.1	S -matrix and T -matrix	499
20.2.2	Interfering scattering amplitudes for identical particles	500
20.2.3	Polarized Fermi gas at low temperature	503
20.2.4	Interference effects in forward and backward scattering	503
20.2.5	Identical spin rotation effect (ISRE)	506
20.2.6	A few examples of effects involving ISRE	508
20.3	The first prediction of BEC in a gas	512
20.3.1	A new derivation of Planck's law for black body radiation	512
20.3.2	Extension of Bose statistics to atomic particles	513
20.3.3	The condensation phenomenon	514
20.3.4	Critical temperature	515
20.3.5	Variation of the number N_0 of condensed atoms with the temperature. Thermodynamic limit	518
20.3.6	Influence of dimensionality	519
20.4	Conclusion	520
21.	The long quest for Bose-Einstein condensation	523
21.1	Introduction	523
21.2	First attempts on hydrogen	524
21.2.1	Spin polarized hydrogen as a quantum gas	524
21.2.2	Production of a spin polarized sample at low temperature	525
21.2.3	Difficulties associated with collisions	526
21.2.4	Need for other methods	527
21.3	Second attempts on hydrogen	527
21.3.1	Wall free confinement. Magnetic trapping	527
21.3.2	Bose-Einstein condensation in a harmonic trap	528

21.3.3	New cooling method: evaporative cooling	529
21.3.4	Need for new detection method of polarized hydrogen . . .	532
21.4	The quest for BEC for alkali atoms	533
21.4.1	Difficulties associated with alkali atoms	533
21.4.2	Advantages of alkali atoms	534
21.5	First observation of Bose-Einstein condensation	535
21.5.1	Time sequence	535
21.5.2	Signature of Bose-Einstein condensation	536
21.5.3	Subsequent observation on hydrogen	538
21.6	Bose-Einstein condensation of other atomic species	538
21.6.1	Experimental improvements	538
21.6.2	Review of new condensates	540
21.7	The first experiments on quantum degenerate Fermi gases	542
21.7.1	Ideal Fermi gas in a three-dimensional harmonic trap . . .	543
21.7.2	Cooling fermions	544
21.7.3	Spatial distribution and Fermi pressure	545
21.7.4	Pairs of fermionic atoms	545
21.8	Conclusion	546
22.	Mean field description of a Bose-Einstein condensate	549
22.1	Introduction	549
22.2	Mean field description of the condensate	550
22.2.1	Variational calculation of the condensate wave function . .	550
22.2.2	Stationary Gross-Pitaevskii equation	551
22.2.3	Expression of the various quantities in terms of the spatial density	552
22.3	Condensate in a box and healing length	553
22.3.1	Condensate in a one-dimensional box	553
22.3.2	Healing length	554
22.4	Condensate in a harmonic trap	555
22.4.1	Total energy and the different interaction regimes	555
22.4.2	Condensate with a positive scattering length and the Thomas-Fermi limit	556
22.5	Condensate with a negative scattering length	559
22.5.1	Condition of stability in 3D	559
22.5.2	Solitonic solution in 1D	560
22.5.3	Collapse and explosion of a condensate in 3D with a negative scattering length	560
22.6	Quantum vortex in an homogeneous condensate	561
22.6.1	Effective Gross-Pitaevskii equation	561
22.6.2	Properties of the velocity field	562
22.7	Time-dependent problems	563

22.7.1	Time-dependent Gross-Pitaevskii equation	563
22.7.2	Analogy with hydrodynamic equations	564
22.7.3	The two contributions to the kinetic energy: Thomas-Fermi approximation for time-dependent problems	565
22.7.4	Harmonic confinement	567
22.8	Conclusion	570
22.9	Appendix: Normal modes of a harmonically trapped condensate	571
22.9.1	Isotropic trap	572
22.9.2	Cylindrically-symmetric trap	575
22.9.3	Scissors mode for anisotropic traps	575
23.	Coherence properties of Bose-Einstein condensates	577
23.1	Introduction	577
23.2	Atomic field operators and correlation functions	579
23.2.1	Brief reminder on second quantization	579
23.2.2	Atomic field operators	580
23.2.3	Examples of physical operators. Field correlation functions .	581
23.2.4	Heisenberg equation of the field operator	583
23.3	Calculation of correlation functions in a few simple cases	583
23.3.1	First-order correlation function for an ideal Bose gas in a box	583
23.3.2	Higher-order spatial correlation functions for an ideal gas of bosons above T_c	586
23.3.3	Correlation functions for a Bose-Einstein condensate . . .	587
23.3.4	A few experimental results	588
23.4	Relative phase of two independent condensates	592
23.4.1	Two condensates in Fock states	593
23.4.2	Phase states	593
23.4.3	Conjugate variable of the relative phase	595
23.4.4	Emergence of a relative phase in an interference experiment	596
23.5	Long range order and order parameter	597
23.5.1	Long range order	597
23.5.2	Order parameter	598
23.6	New effects in atom optics due to atom-atom interactions	599
23.6.1	Collapse and revival of first-order coherence due to interactions	599
23.6.2	An example of nonlinear effects in atom optics: Four-wave mixing with matter waves	601
23.7	Conclusion	602

24. Elementary excitations and superfluidity in Bose-Einstein condensates	603
24.1 Introduction	603
24.2 Bogolubov approach for an homogeneous system	605
24.2.1 Second quantized Hamiltonian	606
24.2.2 Bogolubov quadratic Hamiltonian	607
24.2.3 Physical discussion	608
24.2.4 Energy of the ground state	611
24.2.5 Extension to inhomogeneous systems	612
24.3 Landau criterion for superfluidity in an homogeneous system	614
24.3.1 Microscopic probe	614
24.3.2 Macroscopic approach	616
24.4 Extension of Landau criterion for a condensate in a rotating bucket	616
24.4.1 The rotating bucket	617
24.4.2 Other possible states of the condensate: quantized vortices	617
24.4.3 Various threshold rotation frequencies	620
24.5 Experimental study of vortices in gaseous condensates	621
24.5.1 Introduction	621
24.5.2 A few experimental results	621
24.5.3 Measuring the angular momentum per atom in a rotating condensate	623
24.5.4 Routes to vortex nucleation	624
24.6 Conclusion	628

Frontiers of atomic physics 631

25. Testing fundamental symmetries. Parity violation in atoms	637
25.1 Introduction	637
25.1.1 Historical perspective	637
25.1.2 Atomic parity violation (APV)	639
25.1.3 Organization of this chapter	641
25.2 The first cesium experiment	641
25.2.1 Principle of the experiment	641
25.2.2 Transition dipole moment	642
25.2.3 Existence of a chiral signal in the re-emitted light	645
25.2.4 Calibration of the parity violation amplitude	647
25.3 Connection between the parity violation amplitude and the parameters of the electroweak theory	648
25.3.1 Non-relativistic limit of the weak interaction Hamiltonian	648
25.3.2 Calculation of the parity violation amplitude	649

25.3.3 Nuclear spin-dependent parity violating interactions. Anapole moment	649
25.4 Survey of experimental results	651
25.4.1 Cesium experiments	651
25.4.2 Experiments using other atoms	652
25.5 Conclusion about the importance of APV experiments	653
25.6 Appendix: Testing time reversal symmetry by looking for electric dipole moments	655
26. Quantum gases as simple systems for many-body physics	659
26.1 Introduction	659
26.2 The double well problem for bosonic gases	661
26.2.1 Introduction	661
26.2.2 The Hubbard Hamiltonian	662
26.2.3 The superfluid regime	662
26.2.4 The insulator regime	665
26.2.5 Connection between the superfluid and insulator regimes	667
26.2.6 Production of Schrödinger cat states when interactions are attractive	668
26.2.7 Controlling the tunnelling rate with a modulation of the difference of the two potential depths	669
26.3 Superfluid-Mott insulator transition for a quantum bosonic gas in an optical lattice	670
26.3.1 Bose Hubbard model	670
26.3.2 Qualitative interpretation of the superfluid-Mott insulator transition	670
26.3.3 Experimental observation	672
26.4 Quantum fermionic gas in an optical lattice	672
26.5 Feshbach resonances and Fermi quantum gases	674
26.5.1 Introduction	674
26.5.2 Brief survey of BCS theory	675
26.5.3 A simple model for the BEC-BCS crossover	682
26.5.4 Experimental investigations	684
26.6 Conclusion	689
27. Extreme light	695
27.1 Introduction	695
27.2 Attosecond science	697
27.2.1 Mechanism of production of attosecond pulses	697
27.2.2 Multiple-cycle laser pulse. Train of attosecond pulses	697
27.2.3 Few-cycle laser pulse. Control of the carrier-envelope phase	699

27.2.4	Attosecond metrology	700
27.2.5	A few applications of attosecond pulses	703
27.3	Ultra intense laser pulses	704
27.3.1	Q-switched lasers	705
27.3.2	Mode locking techniques	706
27.3.3	Chirped pulse amplification	709
27.3.4	A few applications of high intensity table-top lasers	709
27.4	Conclusion	713
28.	General conclusion	715
	<i>Bibliography</i>	719
	<i>Index</i>	751