

## CONTENTS

Glossary	xxiii
<b>1 Overview of Microcavities</b>	1
1.1 Properties of microcavities	2
1.1.1 $Q$ -factor and finesse	2
1.1.2 Intracavity field enhancement and field distribution	3
1.1.3 Tuneability and mode separation	3
1.1.4 Angular mode pattern	4
1.1.5 Low-threshold lasing	4
1.1.6 Purcell factor and lifetimes	5
1.1.7 Strong vs. weak coupling	5
1.2 Microcavity realisations	6
1.3 Planar microcavities	6
1.3.1 Metal microcavities	8
1.3.2 Dielectric Bragg mirrors	9
1.4 Spherical mirror microcavities	10
1.5 Pillar microcavities	12
1.6 Whispering-gallery modes	15
1.6.1 Two-dimensional whispering galleries	16
1.6.2 Three-dimensional whispering-galleries	18
1.7 Photonic-crystal cavities	19
1.7.1 Random lasers	19
1.8 Material systems	19
1.8.1 GaN microcavities	21
1.8.2 ZnO microcavities	21
1.8.3 Organic microcavities	21
1.8.4 Transition metal chalcogenides	22
1.8.5 Plasmonic nanocavities	22
1.9 Microcavity lasers	23
1.10 Conclusion	24
<b>2 Classical description of light</b>	25
2.1 Free space	26
2.1.1 Light-field dynamics in free space	26
2.2 Propagation in crystals	29
2.2.1 Plane waves in bulk crystals	29
2.2.2 Absorption of light	33
2.2.3 Kramers–Kronig relations	34

2.3	Coherence	34
2.3.1	Statistical properties of light	34
2.3.2	Spatial and temporal coherence	35
2.3.3	Wiener–Khinchin theorem	40
2.3.4	Hanbury Brown–Twiss effect	43
2.4	Polarisation-dependent optical effects	45
2.4.1	Birefringence	45
2.4.2	Magneto-optical effects	46
2.5	Propagation of light in multilayer planar structures	48
2.6	Photonic eigenmodes of planar systems	52
2.6.1	Photonic bands of 1D periodic structures	54
2.7	Planar microcavities	61
2.8	Tamm plasmons and photonic Tamm states	67
2.9	Stripes, pillars and spheres: photonic wires and dots	69
2.9.1	Cylinders and pillar cavities	71
2.9.2	Spheres	74
2.10	Further reading	77
3	<b>Quantum description of light</b>	79
3.1	Pictures of quantum mechanics	80
3.1.1	Historical background	80
3.1.2	Schrödinger picture	80
3.1.3	Antisymmetry of the wavefunction	89
3.1.4	Symmetry of the wavefunction	90
3.1.5	Heisenberg picture	91
3.1.6	Dirac (interaction) picture	96
3.2	Other formulations	98
3.2.1	Density matrix and Liouvillian	98
3.2.2	Second quantisation	101
3.2.3	Quantisation of the light field	104
3.3	Quantum states	105
3.3.1	Fock states	105
3.3.2	Coherent states	105
3.3.3	Glauber–Sudarshan representation	107
3.3.4	Thermal states	108
3.3.5	Mixture states	109
3.3.6	Power spectrum	110
3.3.7	$g^{(2)}$ and other Glauber correlators	113
3.3.8	Polarisation	119
3.4	Outlook on quantum mechanics for microcavities	121
3.5	Further reading	122

4	<b>Semiclassical description of light–matter coupling</b>	123
4.1	Light–matter interaction	124
4.1.1	Classical limit	124
4.1.2	Einstein coefficients	126
4.2	Optical transitions in semiconductors	129
4.3	Excitons in semiconductors	133
4.3.1	Frenkel and Wannier–Mott excitons	133
4.3.2	Excitons in confined systems	137
4.3.3	Quantum wells	137
4.3.4	Quantum wires and dots	141
4.4	Exciton–photon coupling	143
4.4.1	Surface polaritons	146
4.4.2	Exciton–photon coupling in quantum wells	148
4.4.3	Exciton–photon coupling in quantum wires and dots	152
4.4.4	Dispersion of polaritons in planar microcavities	156
4.4.5	Motional narrowing of cavity polaritons	166
4.4.6	Microcavities with quantum wires or dots	170
5	<b>Quantum description of light–matter coupling</b>	175
5.1	Historical background	176
5.2	Rabi dynamics	176
5.3	Bloch equations	179
5.4	Full quantum picture	181
5.4.1	Light–matter interaction Hamiltonian	182
5.4.2	Dressed bosons	184
5.4.3	Josephson coupling	191
5.4.4	Jaynes–Cummings model	195
5.4.5	Dicke model	202
5.5	Lindblad dissipation	203
5.6	Quantum dynamics with decay and pumping	209
5.6.1	Single-time dynamics of coupled Bose fields	210
5.6.2	Two-time dynamics of coupled Bose fields	216
5.6.3	The two-level system coupled to a Bose field	222
5.7	Excitons in semiconductors	230
5.7.1	Quantisation of the exciton field	230
5.7.2	Excitons as bosons	232
5.7.3	Excitons in quantum dots	232
5.8	Exciton–photon coupling	239
5.8.1	Polariton splitting	240
5.8.2	The polariton Hamiltonian	241
6	<b>Weak-coupling microcavities</b>	243
6.1	Purcell effect	244
6.1.1	The physics of weak coupling	244

6.1.2	Spontaneous emission	245
6.1.3	Quantum dots, 2D excitons and 2D electron–hole pairs	247
6.1.4	Fermi’s golden rule	248
6.1.5	Dynamics of the Purcell effect	251
6.1.6	Experimental realisations	254
6.2	Lasers	256
6.2.1	The physics of lasers	257
6.2.2	Semiconductors in laser physics	261
6.2.3	Vertical-cavity surface-emitting lasers	264
6.2.4	Resonant-cavity LEDs	267
6.2.5	Quantum theory of the laser	268
6.3	Nonlinear optical properties of weak-coupling microcavities	273
6.3.1	Bistability	274
6.3.2	Phase matching	276
6.4	Conclusion	276
7	<b>Strong coupling: resonant effects</b>	277
7.1	Optical properties: background	278
7.1.1	Quantum well microcavities	278
7.1.2	Variations on a theme	279
7.1.3	Motional narrowing	282
7.1.4	Ultra-strong coupling in THz cavities	283
7.1.5	Polariton emission	284
7.2	Near-resonant-pumped optical nonlinearities	285
7.2.1	Pulsed stimulated scattering	285
7.2.2	Quasimode theory of parametric amplification	290
7.2.3	Microcavity parametric oscillators	292
7.3	Resonant excitation case and parametric amplification	294
7.3.1	Semiclassical description	295
7.3.2	Stationary solution and threshold	296
7.3.3	Theoretical approach: quantum model	297
7.3.4	Three-level model	298
7.3.5	Threshold	301
7.4	Two-beam experiment	301
7.4.1	One-beam experiment and spontaneous symmetry breaking	301
7.4.2	Dressing of the dispersion induced by polariton condensates	303
7.4.3	Bistable behaviour	303
7.5	Propagation of polaritons	305
7.5.1	Polariton wavepackets	305
7.5.2	Self-accelerating and self-interfering wavepackets	306
7.5.3	Superfluid propagation	312
7.5.4	Elementary excitation of resonantly pumped polaritons	315
7.5.5	Conventional and unconventional polariton superfluidity	316
7.5.6	High-density effects: the polariton backjet	317

8	<b>Strong coupling: polariton Bose condensation</b>	319
8.1	Introduction	320
8.2	Basic ideas about Bose–Einstein condensation	321
8.2.1	Einstein proposal	321
8.2.2	Experimental realisation	322
8.2.3	Modern definition of Bose–Einstein condensation	324
8.3	Specificities of excitons and polaritons	324
8.3.1	Thermodynamic properties of cavity polaritons	326
8.3.2	Interacting bosons and Bogoliubov model	327
8.3.3	Polariton superfluidity	329
8.3.4	Quasicondensation and local effects	332
8.4	Kinetics of formation of polariton condensates: semiclassical picture	334
8.4.1	Qualitative features	335
8.4.2	The semiclassical Boltzmann equation	338
8.4.3	Numerical solution of Boltzmann equations: practical aspects	339
8.4.4	Effective scattering rates	340
8.4.5	From thermodynamic to kinetic regime	341
8.5	Kinetics of formation of polariton condensates: quantum picture in the Born–Markov approximation	342
8.5.1	Density matrix dynamics of the ground state	344
8.5.2	Discussion	348
8.5.3	Coherence dynamics	349
8.6	Kinetics of formation of polariton condensates: quantum picture beyond the Born–Markov approximation	349
8.6.1	Two-oscillator toy theory	349
8.6.2	Coherence of polariton laser emission	359
8.6.3	Numerical simulations	363
8.6.4	Order parameter and phase diffusion coefficient	364
8.7	Spatial dynamics of polariton condensates	367
8.7.1	Gross–Pitaevskii equation	367
8.7.2	Modified Gross–Pitaevskii equations	368
8.7.3	Bogolon dispersion	369
8.7.4	Spatial coherence. The thermal fluctuation effect	371
8.8	Experiments on Bose–Einstein condensation, superfluidity and lasing of polaritons	372
8.8.1	Experimental observation	372
8.8.2	Polariton lasing vs Bose–Einstein condensation	374
8.8.3	Polariton diodes	374
8.8.4	Experiments on superfluidity	375
8.9	Polariton billiard	376
8.10	Superconductivity mediated by exciton-polaritons	376
8.11	Further reading	379

<b>9 Spin and polarisation</b>	381
9.1 Introduction	382
9.2 Spin relaxation of electrons, holes and excitons in semiconductors	382
9.3 Microcavities in the presence of a magnetic field	387
9.4 Resonant Faraday rotation	388
9.5 Spin relaxation of exciton-polaritons in microcavities: experiment	391
9.6 Spin relaxation of exciton-polaritons in microcavities: theory	396
9.7 Optical spin Hall effect	399
9.8 Full Poincaré beams and polarisation shaping in microcavities	401
9.9 Optically-induced Faraday rotation	404
9.10 Interplay between spin and energy relaxation of exciton-polaritons	406
9.11 Polarisation of Bose condensates and polariton superfluids	410
9.12 Magnetic-field effect and superfluidity	414
9.13 Finite temperature case	418
9.14 Stationary states of spinor condensates	420
9.15 Conclusion	421
9.16 Further reading	421
<b>10 Quantum fluids of light</b>	423
10.1 Introduction	424
10.2 Topological excitations in quantum fluids of light	425
10.2.1 Topological defects in scalar condensates	425
10.2.2 Interaction with a static defect; superfluidity and topology	432
10.3 Half-integer topological defects in spinor quantum fluids	434
10.3.1 Introduction	434
10.3.2 Half-vortices	436
10.3.3 Half-solitons	440
10.4 Hydrodynamic generation of oblique half-solitons and half-vortices	445
10.5 Spin bifurcation theory (broken parity)	449
10.5.1 Paramagnetic solutions.	451
10.5.2 Ferromagnetic solutions.	451
10.6 Engineering of the polariton band structure	452
10.6.1 Introduction	452
10.6.2 Wire cavities	453
10.6.3 Single pillars and molecules	454
10.6.4 Lattices: a few basics about 1D lattices	457
10.6.5 Bright- and gap-solitons in 1D polariton systems	459
10.6.6 Honeycomb lattice (scalar approximation)	461
10.6.7 Honeycomb lattice (polarised)	462
10.6.8 Polariton topological insulators	465
10.7 Further reading	472
<b>11 Quantum polaritonic</b>	475
11.1 Microcavity QED	476

<b>11.1.1 Quantum vs classical polaritons</b>	476
11.1.2 Control of polariton Rabi oscillations	478
11.1.3 Polariton squeezing	480
11.1.4 Polariton statistics	481
11.1.5 Polariton entanglement	483
<b>11.2 Polariton blockade</b>	484
11.2.1 Jaynes–Cummings blockade	485
11.2.2 Kerr blockade	487
11.2.3 Unconventional blockade	488
<b>11.3 Frequency-resolved photon correlations</b>	490
11.3.1 Photo-detection theory	490
11.3.2 The sensor method	492
11.3.3 Two-photon spectra	494
<b>11.4 N-photon emitters</b>	497
11.4.1 Super-Rabi oscillations	498
11.4.2 Robust Jaynes–Cummings resonances	498
11.4.3 Bundles of photons	500
11.4.4 Yudson representation	501
<b>11.5 Exciting with quantum light</b>	502
11.5.1 Cascaded formalism	502
11.5.2 Exciting simple targets	504
11.5.3 Mollow spectroscopy	507
<b>11.6 Quantum information processing</b>	508
11.6.1 Quantum computation	508
11.6.2 Limits of quantum computation	510
11.6.3 Quantum annealing	512
11.6.4 Polariton simulator	514
11.6.5 Other paradigms	516
11.7 Future reading	518
<b>12 Polariton devices</b>	519
<b>12.1 Polariton lasers</b>	520
12.1.1 Concept of polariton lasing	520
12.1.2 Realisation of polariton lasers in semiconductor microcavities	523
<b>12.2 Polariton lasers with electrical injection</b>	523
12.2.1 Experimental manifestations	524
12.2.2 Weak lasing	525
<b>12.3 Polariton terahertz lasers</b>	526
12.3.1 Variety of proposals	526
12.3.2 Polariton terahertz lasers with two-photon excitation	527
12.3.3 Superradiant emission of terahertz radiation by dipolaritons	529
<b>12.4 Bosonic cascade lasers</b>	532
12.4.1 The Boltzmann dynamics of bosonic cascades	532
12.4.2 Quantum model of a bosonic cascade laser	535

12.5	Spatial dynamics of polariton lasing structures	536
12.5.1	Pattern formation	536
12.5.2	Control of lasing modes in structured potentials	537
12.5.3	Bistability and polariton condensate memories	539
12.5.4	Polariton quantum random number generators	539
12.6	Polariton condensate transistors and optical circuits	540
12.6.1	Polariton transistors	542
12.6.2	Polariton neurons	544
12.7	Conclusions	544
12.8	Further reading	545
A	<b>Scattering rates of polariton relaxation</b>	547
A.1	Polariton–phonon interaction	547
A.1.1	Interaction with longitudinal optical phonons	548
A.1.2	Interaction with acoustic phonons	549
A.2	Polariton–electron interaction	550
A.3	Polariton–polariton interaction	552
A.3.1	Polariton decay	552
A.4	Polariton–structural-disorder interaction	553
B	<b>Derivation of the Landau criterion of superfluidity and Landau formula</b>	555
C	<b>Landau quantisation and renormalisation of Rabi splitting</b>	557
<b>Bibliography</b>		561
<b>Index</b>		583