Contents

PREFACE TO THE DOVER EDITION vii

PREFACE xi

LIST OF FREQUENTLY USED SYMBOLS xxiii

1 INTRODUCTION 1

1.1 The helium liquids 1
1.2 Early history of superfluid ^{3}He 5
1.3 Elementary discussion of superfluid ^{3}He 7
1.3.1 The internal structure of Cooper pairs 8
1.3.2 Broken symmetry and the order parameter 9
1.3.3 Orientational effects 12
1.3.4 Textures 15
1.3.5 Superfluid mass currents in ^{3}He-A 16
1.3.6 Dynamic properties 17
1.4 Relation to other fields 21
1.5 Reviews and introductory articles on superfluid ^{3}He 22
1.5.1 General reviews 22
1.5.2 Reviews of specific topics 23
1.5.3 Discussions in books on related subjects 24
1.5.4 Introductory articles 24

2 THEORY OF NORMAL FERMI LIQUIDS 25

2.1 The quasiparticle concept 25
2.2 Thermodynamic properties 28
2.2.1 Entropy and specific heat 28
2.2.2 Spin susceptibility 30
2.2.3 Compressibility 30
2.3 Transport properties 32
2.3.1 Hydrodynamic equations 32
2.3.2 Kinetic equation 33
2.3.3 Quasiparticle lifetime 34
2.3.4 Local equilibrium 37
2.3.5 The linearized collision integral 38
2.3.6 Calculation of the transport coefficients 40

2.4 Collective modes 42
2.5 Theories beyond the Fermi-liquid model 46
2.5.1 Microscopic theories 46
2.5.2 Polarization potential theory 48
2.5.3 Extended Fermi-liquid theory 49
2.5.4 Lattice-gas model 52
2.5.5 Phenomenological approaches 55
2.5.6 Pair interaction 57
Further reading 59

3 PAIR CORRELATIONS IN THE WEAK-COUPLING LIMIT 61

3.1 Cooper instability 61
3.2 Generalized pairing theory 64
3.2.1 Generalized BCS wave function 65
3.2.2 Diagonalization of the mean-field Hamiltonian 67
3.2.3 Single-particle excitations 69
3.2.4 Singlet versus triplet pairing 70
3.3 Pairing theory at finite temperature 71
3.3.1 Effective Hamiltonian 72
3.3.2 Gap equation 73
3.3.3 Critical temperature 75
3.3.4 Free energy 76
3.4 Thermodynamic properties of model states 80
3.4.1 p-wave pairing; the BW and ABM states 80
3.4.2 Gap parameter 82
3.4.3 Specific heat 83
3.4.4 Normal-fluid density 84
3.4.5 Spin susceptibility 87
3.5 Expectation value of two-particle quantities 90
Further reading 92

4 BASIC EXPERIMENTAL PROPERTIES 93

4.1 Experimental techniques for attaining ultralow temperatures 93
4.2 Thermodynamic properties 95
4.2.1 Phase diagram 95
4.2.2 Specific heat 98
4.3 Magnetic properties 100
4.4 Sound propagation 102
4.5 Hydrodynamic properties and superfluidity 106
4.5.1 Normal-fluid density and viscosity 106
4.5.2 Persistent currents 106
4.6 Rotating superfluid ^{3}He 109
Further reading 111

5 PAIR CORRELATIONS BEYOND WEAK COUPLING 113

5.1 General Ginzburg–Landau expansion of the free energy 113
5.2 Phenomenological classification of model states 118
5.2.1 Unitary states 118
5.2.2 Relative stability of model states 122
5.3 Spin fluctuations and the stability of the ABM state 123
5.4 Effects of a magnetic field 129
Further reading 136

6 BROKEN SYMMETRIES AND MACROSCOPIC ORDER 137

6.1 Broken symmetries and group theory 141
6.1.1 The symmetry group describing superfluid ^{3}He 143
6.1.2 A two-dimensional model of superfluid ^{3}He 144
6.1.3 Remaining symmetries of the ^{3}He-A, -A_1 and -B order parameters 147
6.2 Symmetry and order-parameter structure 149
6.2.1 Continuous symmetries 150
6.2.2 Discrete symmetries 162
6.2.3 Symmetry and stationary points of a free energy 165
6.2.4 Symmetry reduction due to a magnetic field or spin–orbit coupling 169
6.2.5 Symmetry classification of an exactly solvable case: d-wave pairing 170
6.2.6 Broken symmetries in high-energy physics 172
6.3 Orientation of the order parameter by internal residual interactions and external fields 174
6.3.1 Dipole interaction 174
6.3.2 Orientation induced by a magnetic field 180
6.3.3 Orientation induced by an electric field 181
6.3.4 Effect of superflow 182
6.3.5 Orientational effects in the B phase due to magnetic and electric fields and superflow 183
6.3.6 Surface energies and boundary conditions 185
Further reading 188

7 SUPERFLOW AND TEXTURES 189

7.1 Superfluidity 190
7.2 Gradient free energy 194
7.2.1 The A phase 195
7.2.2 The B phase 199
7.2.3 Healing lengths 200

7.3 Supercurrents 202
7.3.1 The A phase 203
7.3.2 Finite normal density at $T = 0$ in the A phase 206
7.3.3 Quantization of circulation 208
7.4 Topological investigation of defects 211
7.5 Linear defects 219
7.5.1 Topological stability 219
7.5.2 Topological properties of B-phase vortices 220
7.5.3 Topological properties of A-phase vortices at small distances 223
7.5.4 Topological properties of A-phase vortices at large distances 231
7.5.5 Symmetry classification of vortices 233
7.5.6 Axisymmetric vortices 235
7.5.7 Energetics of vortices 238
7.5.8 Vortices in the A phase 240
7.5.9 Vortices in the B phase 245
7.5.10 Magnetic properties of vortices in the B phase 252
7.6 Rotating superfluid ^{3}He 257
7.6.1 Isotropic superfluid 257
7.6.2 Superfluid ^{3}He 259
7.7 Point defects 263
7.7.1 The B phase 264
7.7.2 The A phase 265
7.8 Planar defects 267
7.8.1 Topological classification 267
7.8.2 Planar solitons in the A phase 272
7.8.3 Planar solitons in the B phase 278
7.9 Surface-induced textures 280
7.9.1 ^{3}He-B in a slab 281
7.9.2 ^{3}He-B in a cylinder 283
7.9.3 ^{3}He-A in a slab 286
7.9.4 ^{3}He-A in a cylinder 288
7.9.5 ^{3}He-A in a sphere 292
7.10 Stability of superflow and related textural transitions in ^{3}He-A 294
7.10.1 Stability of superflow in the bulk liquid 295
7.10.2 The helical instability of the uniform $\hat{\boldsymbol{l}}$ texture 302
7.10.3 Textures and superflow in the presence of boundaries 312
7.10.4 The effect of superflow on domain walls 313
7.10.5 Flow-induced dynamical textures 319
7.11 Pair-breaking critical currents 321
7.11.1 The B phase 323
7.11.2 The A phase 326
7.12 Dissipation of superflow 326
7.12.1 Superfluid ^{4}He 327
7.12.2 The B phase 329
7.12.3 The A phase 331
Further reading 337

8 SPIN DYNAMICS 339

8.1 Derivation of the equations of motion 340
8.2 Nuclear magnetic resonance under linear spatially homogeneous conditions 342
8.2.1 Limit of zero magnetic field 344
8.2.2 Effect of the magnetic field on NMR 345
8.3 Nonlinear NMR phenomena in uniform textures 348
8.3.1 The Leggett equations for the A and B phases 348
8.3.2 Spin dynamics in zero magnetic field 349
8.3.3 Nonlinear spin dynamics in the A phase 355
8.3.4 Nonlinear spin dynamics in the B phase 359
8.4 Texture-induced magnetic resonance phenomena 362
8.4.1 Tilted uniform textures 363
8.4.2 NMR shifts induced by superflow 364
8.4.3 General theory of texture-induced NMR shifts in the A phase 365
8.4.4 Dynamics of solitons in the A phase 376
8.4.5 NMR signature of solitons in the A phase 377
8.4.6 General theory of texture-induced NMR shifts in the B phase 383
8.5 NMR in rotating superfluid ^{3}He 389
8.5.1 The A phase 389
8.5.2 The B phase 390
8.6 Spin-relaxation phenomena 394
Further reading 403

9 HYDRODYNAMIC THEORY 405

9.1 General principles 406
9.1.1 Thermodynamic identities 406
9.1.2 Standard procedure for deriving hydrodynamic equations 407
9.1.3 Equations of motion for the symmetry variables 409
9.2 Hydrodynamic equations for the B phase 411
9.2.1 The thermodynamic identity and equilibrium conditions 412
9.2.2 Hydrodynamic equations for the symmetry variables 413
9.2.3 Hydrodynamic equations for the conserved quantities 414
9.3 Hydrodynamic equations for the A phase 416
9.3.1 The concept of an intrinsic angular momentum 417
9.3.2 Equations of motion for the symmetry variables 418
9.3.3 Thermodynamic identities and equilibrium conditions 420
9.3.4 Entropy production 423
9.3.5 Derivation of the hydrodynamic currents 425
9.3.6 Further consequences of an intrinsic orbital angular momentum 427

9.4 Hydrodynamics for finite magnetic field 429
9.4.1 The A phase in a magnetic field 430
9.4.2 The B phase in a magnetic field 432
Further reading 434

10 TRANSPORT PROPERTIES 435

10.1 Kinetic equations 437
10.1.1 Matrix kinetic equation 438
10.1.2 Conserved quantities and conservation laws 440
10.1.3 Kinetic equation for Bogoliubov quasiparticles 442
10.1.4 Gauge transformation of the kinetic equation 445
10.1.5 Gauge-invariant densities and currents 447
10.2 Collision integral 450
10.2.1 Matrix operator for binary collision processes 450
10.2.2 Conservation properties of the collision integral 453
10.2.3 Collision integral for Bogoliubov quasiparticles 454
10.2.4 Relaxation-time approximation 456
10.2.5 Bogoliubov-quasiparticle relaxation rate 457
10.3 Transport coefficients 461
10.3.1 Transport coefficients of the B phase 464
10.3.2 Intrinsic spin relaxation 475
10.3.3 Transport coefficients of the A phase 479
10.4 Flow in restricted geometries 484
10.4.1 Slip correction to hydrodynamics 484
10.4.2 Andreev reflection 486
10.4.3 Poiseuille flow 489
10.4.4 Vibrating-wire experiments 491
10.4.5 Sound propagation and other flow problems 491
10.4.6 Superfluidity in ^{3}He films 494
10.4.7 Ion mobility 496
Further reading 499

11 COLLECTIVE MODES 501

11.1 Hydrodynamic modes 502
11.1.1 Sound modes 502
11.1.2 Spin-wave modes 505
11.1.3 Other hydrodynamic modes 511
11.2 Symmetry classification of order-parameter modes in the collisionless regime 511
11.2.1 The B phase 512
11.2.2 The A phase 515
11.3 Time-dependent mean-field theory in the collisionless regime 519
11.3.1 Collective modes in the B phase 523
11.3.2 Collective modes in the A phase 526
11.3.3 Effect of residual interactions 533

11.3.4 Observability of collective modes 534
11.4 Collisionless sound 535
11.4.1 Phenomenological model 536
11.4.2 Sound propagation in the B phase 538
11.4.3 Sound propagation in the A phase 543
11.4.4 Nonlinear effects of sound propagation 546
11.4.5 Transverse sound and spin waves 546
Further reading 547

12 AMPLIFICATION OF WEAK INTERACTION EFFECTS DUE TO MACROSCOPIC QUANTUM COHERENCE 549

12.1 Properties of the BCS pair wave function 550
12.2 The permanent orbital magnetic moment of the A phase 553
12.3 The permanent electric dipole moment of the B phase 555
Further reading 560

REFERENCES 561

AUTHOR INDEX 595

SUBJECT INDEX 607