

Table of Contents

1. Relativistic Transformations. The Lorentz Group	1
1.1 Rotations, and Space and Time Reversal for Particles with Spin	1
1.2 Galilean Transformations	4
1.3 Lorentz Transformations. Normal Parameters	6
1.4 Minkowski Space. The Full Lorentz Group	8
1.5 The Lorentz Group	11
1.6 Geometry of Minkowski Space	14
1.7 Transformation Properties of Physical Quantities Under the Lorentz Group	17
1.8 Covariant Form of the Maxwell Equations	19
1.9 Minkowski Space: Metric, Conventions	20
2. The Klein-Gordon Equation. Relativistic Equation for Spinless Particles	23
2.1 The Klein-Gordon Equation. Generalities	23
2.2 Wave Equation for Free Spinless Particles	24
2.3 Plane Waves. Current. Scalar Product	27
2.4 Interaction with the Classical Electromagnetic Field. Gauge Invariance	31
2.5 Particle in a Coulomb Field	32
3. Spin 1/2 Particles	35
3.1 The Dirac Equation	35
3.2 Invariance Properties of the Dirac Equation	37
3.2.1 Rotations	37
3.2.2 Boosts	38
3.2.3 Parity	41
3.2.4 Time Reversal	42
3.3 Density of Particles. Current. Scalar Product	42
3.4 Minimal Replacement. Gauge Invariance. Large and Small Components: Nonrelativistic Limit of the Dirac Equation . . .	45
3.5 Plane Waves. States with Well-Defined Spin	47
3.6 Radial Form of the Dirac Equation. Free-Particle Solutions . .	51

3.6.1	Radial Form of H	52
3.6.2	Free Particles	55
3.7	The Problem of Negative Energies in the Dirac Equation. The Dirac Sea. Hole Theory. Charge Conjugation	57
3.7.1	Negative Energies. The Dirac Sea. Holes	57
3.7.2	Charge Conjugation	58
3.8	Covariants and Projectors	60
3.8.1	Covariants	60
3.8.2	Projectors	61
3.9	Massless Spin 1/2 Particles	62
4.	Dirac Particle in a Potential	67
4.1	Dirac Particle in a Spherical Well	67
4.2	Particle in a Coulomb Potential: Continuum States	69
4.3	Scattering States. Phase Shifts. Cross-sections. Wave Function at the Origin	73
4.3.1	Scattering States. Phase Shifts	73
4.3.2	Cross-sections	74
4.3.3	Wave Function at the Origin	75
4.4	Bound States in a Coulomb Potential	76
4.5	Semirelativistic Approximation: the Foldy-Wouthuysen Transformation	80
4.5.1	General Method	80
4.5.2	Electromagnetic Interactions	84
4.5.3	Free Particle	85
5.	Massive Particles with Spin 1. Massless Spin 1 Particle: Photon Wave Functions. Particles with Higher Spins (3/2, 2, ...)	89
5.1	Particle with Spin 1 and Mass $m \neq 0$	89
5.2	Particle with Spin 1 and Zero Mass: The Photon. Plane Waves. Photon Spin	92
5.2.1	Photon Wave Function. Gauge Fixing. Transformation Properties	92
5.2.2	Plane Waves. Helicity States	95
5.2.3	Field Variables as Wave Functions for the Photon. The Schwinger Gauge	96
5.3	Angular Momentum Eigenstates for the Photon. Vector Spherical Harmonics. Multipoles	97
5.3.1	General Useful Formulas	97
5.3.2	Multipoles	100
5.3.3	Photon Wave Functions with Well-Defined Angular Momentum	102
5.4	Particles with Higher Spins. Rarita-Schwinger and Bargmann-Wigner Equations. The Graviton	104

5.4.1	Rarita-Schwinger Equations	104
5.4.2	Bargmann-Wigner Equations	105
5.4.3	The Graviton	106
6.	General Description of Relativistic States	109
6.1	Preliminaries	109
6.2	Relativistic One-Particle States: General Description	112
6.3	Relativistic States of Massive ($m \neq 0$) Particles	116
6.4	Massless Particles	119
6.5	Many-Particle States. Creation-Annihilation Operators. Fock Space	123
6.6	Connection with the Wave Function Formalism	125
7.	General Description of Relativistic Collisions: S Matrix, Cross-sections and Decay Rates. Partial Wave Analyses ..	129
7.1	Two-Particle States. Separation of the Centre of Mass Motion. States with Well-Defined Angular Momentum	129
7.2	Kinematics of Two-Particle Collisions	131
7.3	The S Matrix. Scattering Amplitude. Nonrelativistic Limit ..	134
7.4	Cross-sections and Decay Rates. The Optical Theorem	136
7.5	Partial Wave Analysis and Phase Shifts. I. Spinless Elastic Scattering. Effective Range Expansion	140
7.5.1	Partial Wave Analysis	140
7.5.2	Effective Range Formalism	143
7.6	Partial Wave Analysis. II. Several Two-Body Channels	143
7.6.1	Multichannel Analysis	143
7.6.2	Effective Range Approximation	146
7.7	Partial Wave Analysis. III. Particles with Spin	146
7.7.1	Spin Analysis	146
7.7.2	Scattering of Spin 0 - Spin 1/2 Particles	148
7.8	Evaluation of the S Matrix	150
7.8.1	The S Matrix and the Interaction Picture	150
7.8.2	The S Matrix in the Lippmann-Schwinger Formalism ..	154
7.8.3	Scattering by Two Interactions	157
8.	Quantization of the Electromagnetic Field. Interaction of Radiation with Matter	159
8.1	Normal, or Wick, Products	159
8.2	Quantization of the Electromagnetic Field (Coulomb Gauge). The Casimir Effect	160
8.2.1	Quantization of the Electromagnetic Field	160
8.2.2	Multipole Expansion	165
8.2.3	The Casimir Effect	166
8.3	Interaction of the Radiation with Slowly Moving Particles ...	169
8.3.1	Radiative Decays, and Absorption of Radiation	171

8.3.2	Low-Energy Compton Scattering	175
8.4	Bremsstrahlung	180
8.5	The Classical Limit. Coherent States	184
8.6	Uncertainty Relations for Field Variables	186
9.	Quantum Fields: Spin 0, 1/2, 1. Covariant Quantization of the Electromagnetic Field	189
9.1	Generalities	189
9.2	Localization of Particles in Relativistic Quantum Mechanics	189
9.3	Retardation and Consistency	191
9.4	Quantization of Scalar Fields and of Massive Vector Fields	193
9.4.1	Second Quantization for Spinless Particles	193
9.4.2	Massive Vector Particles	198
9.5	Quantization of the Dirac Field. Weyl and Majorana Particles	198
9.6	Covariant Quantization of the Electromagnetic Field	204
9.6.1	The Gupta-Bleuler Space	205
9.6.2	Covariant Transformation	210
9.6.3	The Metric Operator Method	212
9.7	Time-Ordered Product. Propagators	213
9.8	Interactions in Quantum Field Theory.	
	Lagrangian Formulation	217
9.8.1	Lagrangian Formalism for Fields	217
9.8.2	Interactions	219
9.9	Gauge Invariance in Quantum Electrodynamics	222
10.	Interactions in Quantum Field Theory.	
	Nonrelativistic Limit. Reduction to Equivalent Potential.	229
10.1	Potentials Equivalent to Field-Theoretic Interactions.	
	General Method	229
10.2	Equivalent Potential for Two Particles	
	in Electromagnetic Interaction	232
10.2.1	Elastic Collision of Two Charged Particles	
	in the Born Approximation	232
10.2.2	Nonrelativistic Limit	236
10.2.3	Relativistic Corrections. The Breit Term	240
10.3	Hydrogenlike Atoms: Hyperfine Structure.	
	System with Two Electrons: the Helium Atom	243
10.3.1	Hydrogenlike Atoms	243
10.3.2	System With Two Electrons. The Helium Atom	244
10.4	Electron-Positron Collisions: Effective Potential. Positronium	247
10.4.1	Scattering Amplitude in the Born Approximation	247
10.4.2	Annihilation Channel	252
10.4.3	Positronium	256
10.5	Scalar and Pseudoscalar Interactions. The Yukawa Potential	258
10.6	Weak Neutral Currents. Parity Violation in Atoms	262

11.	Relativistic Collisions in Field Theory.	
	Feynman Rules. Decays	267
11.1	Electron-Positron Annihilation into Two Photons, and Pair Creation by Two-Photon Collisions	267
11.1.1	e^+e^- Annihilation into 2γ	267
11.1.2	Creation of an e^+e^- Pair by Two Photons	270
11.2	Feynman Rules. Gauge Invariance	271
11.2.1	Feynman Rules for the Evaluation of Transition Amplitudes	271
11.2.2	Gauge Invariance	276
11.3	Polarized and Unpolarized Cross-sections. Sums Over Polarizations	279
11.4	Compton Scattering (Relativistic)	281
11.5	Decay of Bound States	284
11.5.1	General Theory	284
11.5.2	Decays of Positronium	286
11.5.3	Decay of Muonium into e^+e^- . Decays of Quarkonium	288
12.	Relativistic Interactions with Classical Sources	291
12.1	Interaction with a Fixed (Classical) Potential	291
12.1.1	Scattering by an External Field	291
12.1.2	Bremsstrahlung	293
12.2	Photon Emission by a Classical Source.	
	The Bloch-Nordsieck Theorem. Classical Limit	294
12.2.1	Classical Radiation	294
12.2.2	Photon Emission by a Classical Current	296
12.2.3	Radiation of Coherent States	300
12.2.4	The Bloch-Nordsieck Theorem	300
12.3	Propagation of an Electron in a Classical Potential.	
	The Proper-Time Method	301
12.3.1	Electron in a Coulomb Potential	302
12.3.2	The Proper-Time Method	304
12.3.3	Dirac Particle in a Constant Field, or in a Plane Wave	307
	Appendices	309
A.1	Spherical Harmonics, Clebsch-Gordan Coefficients, Matrix Representations of the Rotation Group	309
A.1.1	Spherical Harmonics	309
A.1.2	Some Specific Values	310
A.1.3	Multiplication Formulas	310
A.1.4	Gegenbauer-like Formulas	310
A.1.5	Spinor and Vector Spherical Harmonics	310
A.1.6	Clebsch-Gordan Coefficients	311
A.1.7	Rotation Matrices	313
A.2	Special Functions	313

A.2.1	Kummer, or Confluent Hypergeometric Functions	313
A.2.2	Bessel Functions	314
A.2.3	Spherical Bessel Functions	315
A.2.4	Bessel Functions of the Second Kind	315
A.2.5	Laguerre Polynomials	315
A.3	Relation Between the Lorentz Group and the Group $SL(2, C)$	316
A.4	γ Matrices	319
A.4.1	The Pauli Realization	321
A.4.2	The Weyl Realization	321
A.4.3	The Majorana Realization	321
A.5	Three Lemmas on T and Wick Exponentials	322
A.6	Physical Quantities	324
A.6.1	SI (Gauss) Units	324
A.6.2	Natural Units: $c = \hbar = 1$	324
A.6.3	Other Relations	325
References	327
Index	329