

---

# CONTENTS

---

Preface	xvii
<b>1 Viscosity</b>	
1.1 Introduction	1
1.2 Transmission of Tangential Forces and Sliding Motion	2
1.3 Transmission of Vorticity through a Fluid	2
1.4 Inertia vs Viscosity, or Convection versus Diffusion: The Reynolds Number	3
1.5 Drag and Propulsion	3
1.6 Flow Separation	4
1.7 Hydrodynamic Instability	4
1.8 Dissipation of Mechanical Energy and Production of Entropy	5
1.9 Historical Effects of Viscosity, versus Immediate Effects	5
<b>2 Stresses Acting on an Element of Fluid</b>	
2.1 Introduction	6
2.2 The State of Stress	7
2.3 The Stress Tensor	9
2.4 Symmetry of the Stress Tensor. Principal Stresses	10
2.5 The Essence of the Stress Tensor	11
2.6 Resultant Force per Unit Volume Due to Non-Uniform Stress	12

2.7 Torque Due to Non-Uniform Stress	14
2.8 Torque Due to a Body Force	15
2.9 Summary	15
2.10 Sample Calculations	16
Exercises	20
<b>3 Kinematics of Rotating and Deforming Flow</b>	
3.1 Introduction	21
3.2 Kinematics of a Simple Shear Flow	22
3.3 General Analysis of Relative Motion of Neighboring Points	24
3.4 Synthesis of a Velocity Field	28
3.5 Summary	30
3.6 Sample Calculations	30
Exercises	34
<b>4 Conservation Equations</b>	
4.1 Introduction	38
4.2 Conservation of Mass. The Continuity Equation	39
4.3 Conservation of Momentum	40
4.4 Conservation of Energy	41
4.5 Conservation Equations in Divergence Form	42
4.6 Splitting the Energy Equation	44
4.7 Pressure, Temperature, and Entropy	45
4.8 Conservation Equations for Finite Control Volumes	47
4.9 Application of Conservation Laws at Boundaries	49
4.10 Summary	50
4.11 Sample Calculations	50
Exercises	52
<b>5 Newtonian Fluids and the Navier–Stokes Equations</b>	
5.1 Introduction	54
5.2 Qualitative Properties of Newtonian Fluids	55
5.3 Quantitative Constitutive Equations for Newtonian Fluids	56
5.4 Heat Conduction	59
5.5 Implications of the Second Law of Thermodynamics	59
5.6 The Navier–Stokes Equations	59
5.7 Empirical Conditions at a Physical Interface	60
5.8 Scope of the Navier–Stokes Equations	61
5.9 Summary	62
5.10 Sample Calculations	63
Exercises	66
<b>6 Physical Properties of Fluids</b>	
6.1 Introduction	68
6.2 Density	69
6.3 Specific Heat	70

6.4 Viscosity	70
6.5 Thermal Conductivity	72
6.6 Thermal Diffusivity and Prandtl Number	73
6.7 Surface Tension	74
<b>7 Flows with Nearly Constant Density and Transport Properties</b>	
7.1 Introduction	76
7.2 Dynamic Effects of Density Variation	77
7.3 Kinematic Effects of Density Variation	77
7.4 Temperature Variations	80
7.5 Anelastic Flow	82
7.6 Regular Perturbation Series to Correct for Small Property Variations	83
7.7 The Boussinesq Equations	84
7.8 The Navier–Stokes Equations for Constant Density and Viscosity	85
7.9 Summary	85
7.10 Sample Calculation	86
Exercises	91
<b>8 Vorticity</b>	
8.1 Introduction	93
8.2 Vorticity and Circulation	94
8.3 Vortex Lines and Tubes	95
8.4 The Rate of Change of Circulation around a Material Loop	96
8.5 The Rate of Change of Vorticity of a Fluid Particle	97
8.6 Rotating Frames of Reference	98
8.7 Diffusion of Vorticity	98
8.8 An Extremal Theorem for Vorticity in Plane Flow	101
8.9 The Motion of Vortex Lines and Filaments	102
8.10 Vorticity in an Inviscid Flow?	104
8.11 Effects of Viscosity on the Evolution of Vortex Lines	104
8.12 Velocity Induced by Idealized Concentrations of Vorticity	105
8.13 Kinematic Consequences of the No-Slip Condition	112
8.14 Simulation of Viscous Diffusion. The Random-Vortex Method	113
8.15 Deformation and Dissipation Associated with Concentrated Vorticity	116
8.16 Summary	117
8.17 Sample Calculations	117
Exercises	123
<b>9 Analytical Solutions of the Full Navier–Stokes Equations</b>	
9.1 Introduction	125
9.2 Viscous Diffusion	126
Diffusing Vortex Sheet	126
Diffusing Line Vortex	129
Flows Induced by a Sliding Plane	130

A Self-Similar Free-Convection Problem	133
Impulsively-Started Couette Flow	134
Flow over an Oscillating Wall. Stokes' Second Problem	137
The Ekman Layer	140
Flows Outside a Spinning Cylinder	141
Flows with Uniform Pressure Gradients	145
9.3 Steady Viscometric Flows	149
Couette Flows and Poiseuille Flows	149
Equilibrium Flow of Two Immiscible Fluids Down an Incline	152
Effects of Sidewalls in Plane Couette Flow	153
9.4 Competition between Convection and Diffusion	155
Diffusing Vortex Sheet with Stretched Vortex Lines	155
The Cross-Stretched Vortex Sheet	157
Diffusion Against a Uniform Flow	157
Axisymmetric Flow Induced by Point Force	160
9.5 Summary	171
9.6 Sample Calculation	171
Exercises	172
<b>10 Numerical Solutions of the Full Navier–Stokes Equations</b>	
10.1 Introduction	177
10.2 Finite-Difference Techniques for Ordinary Differential Equations with Two-Point Boundary Conditions	178
Linear, Second-Order Equation	179
Non-Linear Second-Order Equation with Boundary Condition at Infinity	183
Coupled Equations	185
Steady Three-Dimensional Stagnation-Point Flows	187
Steady Flow Induced by a Spinning Disk	193
10.3 Flows Governed by Parabolic Partial Differential Equations	198
Flow Outside an Impulsively Spun-Up Cylinder	198
Marching Forward in Time. Computational Instability	200
Transient Development of the Spinning-Disk Flow	204
Transient Three-Dimensional Stagnation-Point Flow	207
10.4 Elliptic Partial Differential Equations	212
Flows with Multi-Dimensional Diffusion	213
10.5 Computation of Enclosed Viscous Flows	218
Wall-Driven Cavity Flow	219
10.6 Computation of External Viscous Flows	220
Flow Past an Impulsively Accelerated Cylinder	221
10.7 Summary	223
Exercises	223
<b>11 Creeping Flows</b>	
11.1 Introduction	225
11.2 General Features of Creeping Flow	227

11.3 Creeping Flow in Slender Layers	229
Order-of-Magnitude Analysis: the Squeezed Film	229
Velocity and Stress Fields for Creeping Flow in Slender Films	232
Reynolds' Equation for the Pressure. Lubrication Theory	234
11.4 Sample Lubrication Problems	235
The Squeezed Film	235
A Slider Bearing	235
A Journal Bearing	240
A Simple Example of Forced Lubrication	244
The Hele–Shaw Cell	246
11.5 Slender Viscous Films with Free Surfaces	247
A Windblown Oil Film	248
Draining Down an Inclined Plane	249
Axisymmetric Pool on a Spinning Disk	250
Gravity-Driven Spreading of a Circular Pool	251
Effects of Surface Tension	252
11.6 Two-Dimensional Creeping Flows in Corners	255
Flow in a Wedge with a Line Source or Sink at the Apex	256
Effect of a Squeegee or a Submerging Belt	256
Flow in a Wedge with Swinging Walls	258
Flow in a Wedge, Driven by Circumferential Motion at Large $r$	258
Summary	264
11.7 External Creeping Flows	266
Stokes' Flow Over a Sphere	266
Flow Induced Outside a Spinning Sphere	272
11.8 Effects of Body Shape on External Creeping Flows	273
Prolate Spheroids	273
Oblate Spheroids	275
Spheroid of Minimum Drag for a Fixed Volume	276
The Spherical Cap	277
The Circular Cylinder Broadside to the Flow. The Stokes' Paradox	278
The Prolate Spheroid, Broadside to the Flow	279
11.9 Singular Solutions for the Creeping-Flow Equations	279
The Stokeslet	280
Other Singular Solutions	281
Techniques of Superposition	282
11.10 Small Effects of Inertia	285
Small Inertial Effects in Slender, Bounded Flows	285
Improved Description of External Flow Far from a Body in Creeping Flow	288
Lift on a Sphere in Shear Flow	295
11.11 Creeping Flow Through a Porous Medium	297
11.12 Summary	298
11.13 Sample Calculations	300
Exercises	305
<b>12 Laminar Boundary Layers</b>	
12.1 Introduction	310
12.2 Boundary-Layer Coordinates and Equations	313

Coordinate Systems	313
The Boundary-Layer Equations	318
Flow Equations for a Slender Axisymmetric Jet	319
12.3 The Momentum-Integral Equation	322
Steady, Plane Flow Over a Solid Wall. Thwaites' Method	323
Sample Applications of Thwaites' Method	325
Some Analyses Assuming Constant Shape Factors	332
12.4 Transformation of the Boundary-Layer Equations. Similiar Solutions	336
A Transformation Based on the Momentum-Integral Equation	339
Self-Similar Boundary Layers	340
Falkner-Skan Flows	341
Sink-Driven Boundary Layers	342
The Vortex-Driven Boundary Layer	343
The Effects of Pressure Gradient and Suction on Profile Shape	343
12.5 Numerical Prediction of the Evolution of Nonsimilar Boundary Layers	345
Example: The Boundary Layer on a Wall Under a Line Vortex	346
Other Examples	354
The Boundary-Layer Equations with Pressure Eliminated	363
An Idealized Entry-Length Problem	363
12.6 Boundary-Layer Control by Distributed Suction	367
12.7 Boundary Layers Within Boundary Layers	371
The Flat Plate with Discontinuous Suction	372
The Wake Behind a Finite Flat Plate	374
Wakes in an Accelerating or Decelerating Stream	376
12.8 Unsteady and/or Three-Dimensional Boundary Layers	377
Propagation of Information in a Boundary Layer	378
12.9 Unsteady, Two-Dimensional, Boundary Layers	380
Impulsively-Started Boundary Layer on a Wedge	381
Impulsively-Started Flows within the Region of Influence of a Leading Edge	381
12.10 Three-Dimensional Boundary Layers	388
Design of a Coordinate System To Cover the Wall	388
Nozzle Sidewall Boundary Layer	393
Impulsively Started Flow on Frontal Symmetry Plane of a Hot Dog	396
12.11 Boundary-Layer Separation	403
Steady, Two-Dimensional Flow	403
Unsteady, Two-Dimensional Flow	407
Three-Dimensional Steady Flow Separation	415
12.12 Higher-Order Boundary-Layer Theory	419
Second-Order Theory for a Parabolic Cylinder	423
Second-Order Analysis of Entry Flow into a Channel	427
12.13 Summary	428
12.14 Sample Calculation	430
Exercises	432
<b>13 Instability of Viscous Flows</b>	
13.1 Introduction	438
13.2 Vorticity and Instability	441

13.3 Examples of Unstable Arrangements of Vorticity	444
Disturbed Equilibrium of a Row of Vortices	444
A Sinusoidally Rippled Vortex Sheet	446
Vorticity Distributed between Coaxial Rotating Cylinders	449
13.4 Mathematical Analysis of the Instability of Steady, Plane Flows	451
Normal-Mode Analysis	452
Definition and Classification of Instabilities	454
Convective Instability and Absolute Instability	455
Global Instability	456
Temporal Instability and Spatial Instability	457
13.5 Analysis of Temporal Instability. Theorems That Narrow the Search for Unstable Basic Flows, and for the Parameters of Dangerous Normal Modes.	459
Rayleigh's and Fjørtoft's Theorems Concerning Inflection Points in the Velocity Profile	459
Howard's Semicircle Theorem	460
Squire's Equivalence Theorem. The Development of Obliquely Propagating Normal Modes	463
13.6 Effects of Wavelength on Stability. Short-Wave Cutoffs	465
Layer of Uniform Vorticity Between Uniform Streams	466
Calculation of the Streamfunction and of Streamlines	468
Evolution of Material Surfaces	469
13.7 Numerical Searches for Eigensolutions	470
Stability of the Velocity Profile $U = \text{erf}(2\eta/\sqrt{\pi})$	471
Stabilizing Effects of Viscosity	474
Other Symmetric Flows. Jets and Wakes	476
Stability of Wall-Bounded Shear Flows	479
The Destabilizing Effect of Viscosity. Tollmien-Schlichting Waves	481
13.8 Collected Results of Normal-Mode Theory for Parallel Shear Flows	487
Evolution of a Tollmien-Schlichting Wave as it Moves Downstream	490
13.9 Centrifugal Instability. Taylor-Görtler Vortices	491
The Instability of Circular Couette Flow. Taylor Vortices	492
Instability of Boundary-Layer Flow along a Concave Wall. Görtler Vortices	497
13.10 Instability of Three-Dimensional Boundary Layers	503
Stability of Flow on Spinning Disk	504
Flow on Other Spinning Bodies	505
13.11 Viscous Fingering	508
Linear Analysis of the Stability of an Initially Plane Interface between Immiscible Fluids	509
Critical Displacement Speed	512
13.12 The Transition to Turbulence. Experiment and Theory	513
Why Do Some Flows become Turbulent, When Theory Predicts Stability?	515
What Kinds of Disturbances Should one Expect to See, When Linera Theory Says that Many Kinds Might Grow?	516
13.13 The Relevance of Stability Theory for the Prediction of Transition	532
Competing Theories	532
Receptivity	533

Transition beyond the Stage of Secondary Instability	534
Turbulent Spots	537
Direct Computational Simulations of Transition	539
A Competing View of the Onset of Complex Motion. Chaos.	540
13.14 Summary	543
Exercises	544
<b>14 Turbulent Flow</b>	
14.1 Introduction	548
Instability and Indeterminacy	548
Mean Flow Behavior	548
Practical Consequences of Turbulence	549
The Roles of Viscosity in Turbulent Flow	550
14.2 The Description of Turbulent Flow	551
Visual Impressions	551
Statistical Description	552
Fourier Analysis of Time Series	553
Energy Spectrum	555
Correlation Functions	555
Scales of Turbulence	557
Intermittency and Conditioned Sampling	558
14.3 Theory of the Mean Flow. Reynolds-Averaged Equations	559
Indeterminacy Again	560
Energy Accounting	561
Production and Dissipation of Turbulence	561
14.4 Deterministic Cartoons of Turbulence	563
The Burgers Vortex	564
Vortex Sheets into Vortex Tubes: The Corcos-Lin Cartoon	567
Generation of New Scales of Motion. Streamwise Vortex in Shear Flow	568
Self-Induced Stretching of a Vortex	574
14.5 Empirical Generalizations for Statistically Stationary Flows	577
14.6 Turbulent Flow Along a Wall	579
The Dual Structure of Turbulent Wall Layers	579
The Law of the Wall	580
Flow in the Outer Layers	588
The Overlap Layer	591
Strong Adverse $dP/dx$ . The Schofield-Perry Velocity-Defect Law	592
Effects of Longitudinal Curvature	594
The Measurement of Turbulent Skin Friction	596
Statistics of Fluctuations in the Outer Regions of Turbulent Wall Flows	599
14.7 Free Turbulent Flows	611
Introduction	611
Plane Mixing Layers	612
Entrainment: Induced Flow Normal to the Layer	614
Reynold Stress Profiles. Spectra	615
Intermittency	615
Mixedness	615
Instantaneous Flow Structures	618

Molecular Mixing	623
Perturbed Mixing Layers	624
Jets and Wakes	625
Bouyant Plumes and Thermals	631
14.8 Numerical Simulations of Turbulent Flow	634
Individual Realizations	634
Statistical Sampling	639
Advantages of a Computationally Generated Data Base	639
Large-eddy Simulations	639
14.9 Summary	640

## Appendix A Mathematical Aids

A. Coordinate-free Representation of Vectors and Tensors	644
B. Representation of Differential Quantities in Orthogonal Coordinate Systems	651
C. Equations of Motion in Rectangular, Cylindrical, and Spherical Coordinates	656

## Appendix B FORTRAN Programs

B1 SHEAR	659
B2 HAMEL	661
B3 EKMAN	663
B4 STAGPT	666
B5 SPNCYL	670
B6 SPNDSK	673
B7 DIRECOU	678
B8 ADICOU	680
B9 NEWBL	683
B10 ENTRY	689
B11 BLCTRL	695
B12 TRANBL	704
B13 SYMMBL	712
B14 HOTDOG	719
B15 EIGEN	727
B16 TAYLOR	729

Index	733
-------	-----