Contents

Preface xxi

Preface to the First Edition xxiii

Introduction 1

Part 1 Nuclear Magnetism 3

1 Matter 5

1.1 Atoms and Nuclei 5
1.2 Spin 5
1.2.1 Classical angular momentum 6
1.2.2 Quantum angular momentum 6
1.2.3 Spin angular momentum 7
1.2.4 Combining angular momenta 8
1.2.5 The Pauli Principle 9
1.3 Nuclei 9
1.3.1 The fundamental particles 9
1.3.2 Neutrons and protons 10
1.3.3 Isotopes 11
1.4 Nuclear Spin 12
1.4.1 Nuclear spin states 12
1.4.2 Nuclear Zeeman splitting 14
1.4.3 Zero-spin nuclei 14
1.4.4 Spin-1/2 nuclei 15
1.4.5 Quadrupolar nuclei with integer spin 15
1.4.6 Quadrupolar nuclei with half-integer spin 15
1.5 Atomic and Molecular Structure 15
1.5.1 Atoms 15
1.5.2 Molecules 16
1.6 States of Matter 17
1.6.1 Gases 17
1.6.2 Liquids 17
1.6.3 Solids 19

2 Magnetism 23

2.1 The Electromagnetic Field 23
2.2 Macroscopic Magnetism 23
2.3 Microscopic Magnetism 25
2.4 Spin Precession 26
2.5 Larmor Frequency 29
2.6 Spin–Lattice Relaxation: Nuclear Paramagnetism 30
2.7 Transverse Magnetization and Transverse Relaxation 33
2.8 NMR Signal 36
2.9 Electronic Magnetism 36

3 NMR Spectroscopy 39

3.1 A Simple Pulse Sequence 39
3.2 A Simple Spectrum 39
3.3 Isotopomeric Spectra 42
3.4 Relative Spectral Frequencies: Case of Positive Gyromagnetic Ratio 44
3.5 Relative Spectral Frequencies: Case of Negative Gyromagnetic Ratio 46
3.6 Inhomogeneous Broadening 48
3.7 Chemical Shifts 50
3.8 *J*-Coupling Multiplets 56
3.9 Heteronuclear Decoupling 59

Part 2 The NMR Experiment 63

4 The NMR Spectrometer 65

4.1 The Magnet 65
4.2 The Transmitter Section 66
4.2.1 The synthesizer: radio-frequency phase shifts 67
4.2.2 The pulse gate: radio-frequency pulses 68
4.2.3 Radio-frequency amplifier 69
4.3 The Duplexer 69
4.4 The Probe 70
4.5 The Receiver Section 72
4.5.1 Signal preamplifier 73
4.5.2 The quadrature receiver 73
4.5.3 Analogue–digital conversion 74
4.5.4 Signal phase shifting 76
4.6 Overview of the Radio-Frequency Section 76
4.7 Pulsed Field Gradients 77
4.7.1 Magnetic field gradients 78
4.7.2 Field gradient coils 79
4.7.3 Field gradient control 80

5 Fourier Transform NMR 85

5.1 A Single-Pulse Experiment 85
5.2 Signal Averaging 86
5.3 Multiple-Pulse Experiments: Phase Cycling 89
5.4 Heteronuclear Experiments 90
5.5 Pulsed Field Gradient Sequences 91
5.6 Arrayed Experiments 91
5.7 NMR Signal 93
5.8 NMR Spectrum 96
5.8.1 Fourier transformation 96
5.8.2 Lorentzians 96
5.8.3 Explanation of Fourier transformation 100
5.8.4 Spectral phase shifts 102
5.8.5 Frequency-dependent phase correction 103
5.9 Two-Dimensional Spectroscopy 105
5.9.1 Two-dimensional signal surface 105
5.9.2 Two-dimensional Fourier transformation 105
5.9.3 Phase twist peaks 107
5.9.4 Pure absorption two-dimensional spectra 109
5.10 Three-Dimensional Spectroscopy 114

Part 3 Quantum Mechanics 119

6 Mathematical Techniques 121

6.1 Functions 121
6.1.1 Continuous functions 121
6.1.2 Normalization 122
6.1.3 Orthogonal and orthonormal functions 122
6.1.4 Dirac notation 122
6.1.5 Vector representation of functions 123
6.2 Operators 125
6.2.1 Commutation 126
6.2.2 Matrix representations 126
6.2.3 Diagonal matrices 129
6.2.4 Block diagonal matrices 129
6.2.5 Inverse 130
6.2.6 Adjoint 130
6.2.7 Hermitian operators 131
6.2.8 Unitary operators 131
6.3 Eigenfunctions, Eigenvalues and Eigenvectors 131
6.3.1 Eigenequations 131
6.3.2 Degeneracy 131
6.3.3 Eigenfunctions and eigenvalues of Hermitian operators 132
6.3.4 Eigenfunctions of commuting operators: non-degenerate case 132
6.3.5 Eigenfunctions of commuting operators: degenerate case 132
6.3.6 Eigenfunctions of commuting operators: summary 133
6.3.7 Eigenvectors 134

6.4 Diagonalization 134
6.4.1 Diagonalization of Hermitian or unitary matrices 135
6.5 Exponential Operators 135
6.5.1 Powers of operators 135
6.5.2 Exponentials of operators 136
6.5.3 Exponentials of unity and null operators 136
6.5.4 Products of exponential operators 137
6.5.5 Inverses of exponential operators 137
6.5.6 Complex exponentials of operators 137
6.5.7 Exponentials of small operators 137
6.5.8 Matrix representations of exponential operators 138
6.6 Cyclic Commutation 138
6.6.1 Definition of cyclic commutation 138
6.6.2 Sandwich formula 139

7 Review of Quantum Mechanics 143

7.1 Spinless Quantum Mechanics 143
7.1.1 The state of the particle 143
7.1.2 The equation of motion 144
7.1.3 Experimental observations 144
7.2 Energy Levels 145
7.3 Natural Units 146
7.4 Superposition States and Stationary States 147
7.5 Conservation Laws 148
7.6 Angular Momentum 148
7.6.1 Angular momentum operators 149
7.6.2 Rotation operators 149
7.6.3 Rotation sandwiches 151
7.6.4 Angular momentum eigenstates and eigenvalues 152
7.6.5 The angular momentum eigenstates 154
7.6.6 Shift operators 154
7.6.7 Matrix representations of the angular momentum operators 156
7.7 Spin 157
7.7.1 Spin angular momentum operators 157
7.7.2 Spin rotation operators 158
7.7.3 Spin Zeeman basis 158
7.7.4 Trace 159
7.8 Spin-1/2 160
7.8.1 Zeeman eigenstates 160
7.8.2 Angular momentum operators 160
7.8.3 Spin-1/2 rotation operators 160
7.8.4 Unity operator 161
7.8.5 Shift operators 161
7.8.6 Projection operators 161
7.8.7 Ket-bra notation 162
7.9 Higher Spin 162
7.9.1 Spin $I = 1$ 163
7.9.2 Spin $I = 3/2$ 164
7.9.3 Higher spins 165

Part 4 Nuclear Spin Interactions 169

8 Nuclear Spin Hamiltonian 171

8.1 Spin Hamiltonian Hypothesis 171
8.2 Electromagnetic Interactions 172
8.2.1 Electric spin Hamiltonian 173
8.2.2 Magnetic spin interactions 176
8.3 External and Internal Spin Interactions 177
8.3.1 Spin interactions: summary 177
8.4 External Magnetic Fields 177
8.4.1 Static field 179
8.4.2 Radio-frequency field 179
8.4.3 Gradient field 181
8.4.4 External spin interactions: summary 181
8.5 Internal Spin Hamiltonian 182
8.5.1 The internal spin interactions 182
8.5.2 Simplification of the internal Hamiltonian 185
8.6 Motional Averaging 186
8.6.1 Modes of molecular motion 186
8.6.2 Molecular rotations 186
8.6.3 Molecular translations 187
8.6.4 Intramolecular and intermolecular spin interactions 189
8.6.5 Summary of motional averaging 190

9 Internal Spin Interactions 195

9.1 Chemical Shift 195
9.1.1 Chemical shift tensor 196
9.1.2 Principal axes 197
9.1.3 Principal values 198
9.1.4 Isotropic chemical shift 198
9.1.5 Chemical shift anisotropy (CSA) 198
9.1.6 Chemical shift for an arbitrary molecular orientation 200
9.1.7 Chemical shift frequency 201
9.1.8 Chemical shift interaction in isotropic liquids 201
9.1.9 Chemical shift interaction in anisotropic liquids 203
9.1.10 Chemical shift interaction in solids 204
9.1.11 Chemical shift interaction: summary 206
9.2 Electric Quadrupole Coupling 206
9.2.1 Electric field gradient tensor 207
9.2.2 Nuclear quadrupole Hamiltonian 208
9.2.3 Isotropic liquids 209
9.2.4 Anisotropic liquids 209
9.2.5 Solids 210
9.2.6 Quadrupole interaction: summary 210
9.3 Direct Dipole–Dipole Coupling 211
9.3.1 Secular dipole–dipole coupling 213
9.3.2 Dipole–dipole coupling in isotropic liquids 215

9.3.3 Dipole–dipole coupling in liquid crystals 216
9.3.4 Dipole–dipole coupling in solids 216
9.3.5 Dipole–dipole interaction: summary 217
9.4 *J*-Coupling 217
9.4.1 Isotropic *J*-coupling 219
9.4.2 Liquid crystals and solids 221
9.4.3 Mechanism of the *J*-coupling 222
9.4.4 *J*-coupling: summary 223
9.5 Spin–Rotation Interaction 223
9.6 Summary of the Spin Hamiltonian Terms 224

Part 5 Uncoupled Spins 229

10 Single Spin-1/2 231

10.1 Zeeman Eigenstates 231
10.2 Measurement of Angular Momentum: Quantum Indeterminacy 232
10.3 Energy Levels 233
10.4 Superposition States 234
10.4.1 General spin states 234
10.4.2 Vector notation 234
10.4.3 Some particular states 235
10.4.4 Phase factors 237
10.5 Spin Precession 238
10.5.1 Dynamics of the eigenstates 239
10.5.2 Dynamics of the superposition states 240
10.6 Rotating Frame 241
10.7 Precession in the Rotating Frame 245
10.8 Radio-frequency Pulse 247
10.8.1 Rotating-frame Hamiltonian 247
10.8.2 *x*-pulse 248
10.8.3 Nutation 251
10.8.4 Pulse of general phase 252
10.8.5 Off-resonance effects 253

11 Ensemble of Spins-1/2 259

11.1 Spin Density Operator 259
11.2 Populations and Coherences 261
11.2.1 Density matrix 261
11.2.2 Box notation 261
11.2.3 Balls and arrows 262
11.2.4 Orders of coherence 263
11.2.5 Relationships between populations and coherences 263
11.2.6 Physical interpretation of the populations 264
11.2.7 Physical interpretation of the coherences 265
11.3 Thermal Equilibrium 266
11.4 Rotating-Frame Density Operator 268

11.5 Magnetization Vector 269
11.6 Strong Radio-Frequency Pulse 270
11.6.1 Excitation of coherence 271
11.6.2 Population inversion 273
11.6.3 Cycle of states 274
11.6.4 Stimulated absorption and emission 275
11.7 Free Precession Without Relaxation 276
11.8 Operator Transformations 279
11.8.1 Pulse of phase $\phi_p = 0$ 279
11.8.2 Pulse of phase $\phi_p = \pi/2$ 279
11.8.3 Pulse of phase $\phi_p = \pi$ 279
11.8.4 Pulse of phase $\phi_p = 3\pi/2$ 279
11.8.5 Pulse of general phase ϕ_p 280
11.8.6 Free precession for an interval τ 280
11.9 Free Evolution with Relaxation 281
11.9.1 Transverse relaxation 281
11.9.2 Longitudinal relaxation 283
11.10 Magnetization Vector Trajectories 285
11.11 NMR Signal and NMR Spectrum 287
11.12 Single-Pulse Spectra 289

12 Experiments on Non-Interacting Spins-1/2 295

12.1 Inversion Recovery: Measurement of T_1 295
12.2 Spin Echoes: Measurement of T_2 298
12.2.1 Homogenous and inhomogenenous broadening 298
12.2.2 Inhomogenenous broadening in the time domain 299
12.2.3 Spin echo pulse sequence 299
12.2.4 Refocusing 302
12.2.5 Coherence interpretation 303
12.2.6 Coherence transfer pathway 305
12.3 Spin Locking: Measurement of $T_{1\rho}$ 305
12.4 Gradient Echoes 306
12.5 Slice Selection 307
12.6 NMR Imaging 309

13 Quadrupolar Nuclei 319

13.1 Spin $I = 1$ 319
13.1.1 Spin-1 states 319
13.1.2 Spin-1 energy levels 320
13.1.3 Spin-1 density matrix 321
13.1.4 Coherence evolution 323
13.1.5 Observable coherences and NMR spectrum 325
13.1.6 Thermal equilibrium 326
13.1.7 Strong radio-frequency pulse 326
13.1.8 Excitation of coherence 328
13.1.9 NMR spectrum 328
13.1.10 Quadrupolar echo 331

13.2 Spin $I = 3/2$ 334
13.2.1 Spin-3/2 energy levels 335
13.2.2 Populations and coherences 336
13.2.3 NMR signal 338
13.2.4 Single pulse spectrum 339
13.2.5 Spin-3/2 spectra for small quadrupole couplings 341
13.2.6 Second-order quadrupole couplings 342
13.2.7 Central transition excitation 343
13.2.8 Central transition echo 345
13.3 Spin $I = 5/2$ 345
13.4 Spins $I = 7/2$ 349
13.5 Spins $I = 9/2$ 350

Part 6 Coupled Spins **353**

14 Spin-1/2 Pairs **355**

14.1 Coupling Regimes 355
14.2 Zeeman Product States and Superposition States 356
14.3 Spin-Pair Hamiltonian 357
14.4 Pairs of Magnetically Equivalent Spins 359
14.4.1 Singlets and triplets 359
14.4.2 Energy levels 360
14.4.3 NMR spectra 362
14.4.4 Dipolar echo 363
14.5 Weakly Coupled Spin Pairs 363
14.5.1 Weak coupling 363
14.5.2 AX spin systems 364
14.5.3 Energy levels 364
14.5.4 AX spectrum 365
14.5.5 Heteronuclear spin pairs 366

15 Homonuclear AX System **369**

15.1 Eigenstates and Energy Levels 369
15.2 Density Operator 370
15.3 Rotating Frame 375
15.4 Free Evolution 376
15.4.1 Evolution of a spin pair 376
15.4.2 Evolution of the coherences 377
15.5 Spectrum of the AX System: Spin–Spin Splitting 378
15.6 Product Operators 381
15.6.1 Construction of product operators 382
15.6.2 Populations and coherences 383
15.6.3 Spin orientations 386
15.7 Thermal Equilibrium 389
15.8 Radio-Frequency Pulses 391
15.8.1 Rotations of a single spin pair 392
15.8.2 Rotations of the spin density operator 393

15.8.3 Operator transformations 395
15.9 Free Evolution of the Product Operators 397
15.9.1 Chemical shift evolution 399
15.9.2 J-coupling evolution 400
15.9.3 Relaxation 405
15.10 Spin Echo Sandwich 405

16 Experiments on AX Systems **409**

16.1 COSY 409
16.1.1 The assignment problem 409
16.1.2 COSY pulse sequence 411
16.1.3 Theory of COSY: coherence interpretation 411
16.1.4 Product operator interpretation 415
16.1.5 Experimental examples 418
16.2 INADEQUATE 418
16.2.1 ^{13}C isotopomers 418
16.2.2 Pulse sequence 423
16.2.3 Theory of INADEQUATE 424
16.2.4 Coherence transfer pathways and phase cycling 429
16.2.5 Two-dimensional INADEQUATE 431
16.3 INEPT 436
16.3.1 The sensitivity of nuclear isotopes 436
16.3.2 INEPT pulse sequence 437
16.3.3 Refocused INEPT 440
16.4 Residual Dipolar Couplings 443
16.4.1 Angular information 443
16.4.2 Spin Hamiltonian 443
16.4.3 Orienting media 444
16.4.4 Doublet splittings 446

17 Many-Spin Systems **453**

17.1 Molecular Spin System 453
17.2 Spin Ensemble 454
17.3 Motionally Suppressed J-Couplings 454
17.4 Chemical Equivalence 455
17.5 Magnetic Equivalence 458
17.6 Weak Coupling 461
17.7 Heteronuclear Spin Systems 462
17.8 Alphabet Notation 463
17.9 Spin Coupling Topologies 464

18 Many-Spin Dynamics **467**

18.1 Spin Hamiltonian 467
18.2 Energy Eigenstates 468

18.3 Superposition States 469
18.4 Spin Density Operator 470
18.5 Populations and Coherences 471
18.5.1 Coherence orders 471
18.5.2 Combination coherences and simple coherences 471
18.5.3 Coherence frequencies 472
18.5.4 Degenerate coherences 473
18.5.5 Observable coherences 473
18.6 NMR Spectra 475
18.7 Many-Spin Product Operators 477
18.7.1 Construction of product operators 477
18.7.2 Populations and coherences 478
18.7.3 Physical interpretation of product operators 480
18.8 Thermal Equilibrium 481
18.9 Radio-Frequency Pulses 481
18.10 Free Precession 482
18.10.1 Chemical shift evolution 482
18.10.2 J-coupling evolution 483
18.10.3 Relaxation 485
18.11 Spin Echo Sandwiches 485
18.12 INEPT in an I_2S System 488
18.13 COSY in Multiple-Spin Systems 491
18.13.1 AMX spectrum 492
18.13.2 Active and passive spins 493
18.13.3 Cross-peak multiplets 494
18.13.4 Diagonal peaks 496
18.13.5 Linear spin systems 497
18.14 TOCSY 497
18.14.1 The ambiguity of COSY spectra 497
18.14.2 TOCSY pulse sequence 499
18.14.3 Theory of TOCSY 499

Part 7 Motion and Relaxation **507**

19 Motion **509**

19.1 Motional Processes 509
19.1.1 Molecular vibrations 509
19.1.2 Local rotations of molecular groups 510
19.1.3 Molecular flexibility 510
19.1.4 Chemical exchange 510
19.1.5 Molecular rotations 511
19.1.6 Translational motion 512
19.1.7 Mechanical motion 513
19.2 Motional Time-Scales 513
19.3 Motional Effects 514
19.4 Motional Averaging 515
19.5 Motional Lineshapes and Two-Site Exchange 516

19.5.1 Slow intermediate exchange and motional broadening 518
19.5.2 Fast intermediate exchange and motional narrowing 520
19.5.3 Averaging of J-splittings 523
19.5.4 Asymmetric two-site exchange 524
19.5.5 Knight shift 525
19.5.6 Paramagnetic shifts 527
19.6 Sample Spinning 527
19.7 Longitudinal Magnetization Exchange 529
19.7.1 Two-dimensional exchange spectroscopy 529
19.7.2 Theory 532
19.7.3 Motional regimes 539
19.8 Diffusion 539

20 Relaxation **543**

20.1 Types of Relaxation 543
20.2 Relaxation Mechanisms 543
20.3 Random Field Relaxation 545
20.3.1 Autocorrelation functions and correlation times 545
20.3.2 Spectral density 548
20.3.3 Normalized spectral density 549
20.3.4 Transition probabilities 550
20.3.5 Thermally corrected transition probabilities 551
20.3.6 Spin–lattice relaxation 552
20.4 Dipole–Dipole Relaxation 556
20.4.1 Rotational correlation time 556
20.4.2 Transition probabilities 557
20.4.3 Solomon equations 561
20.4.4 Longitudinal relaxation 564
20.4.5 Transverse relaxation 565
20.5 Steady-State Nuclear Overhauser Effect 566
20.6 NOESY 570
20.6.1 NOESY pulse sequence 570
20.6.2 NOESY signal 570
20.6.3 NOESY spectra 573
20.6.4 NOESY and chemical exchange 575
20.6.5 Molecular structure determination 576
20.7 ROESY 577
20.7.1 Transverse cross-relaxation 577
20.7.2 Spin locking 578
20.7.3 Transverse Solomon equations 578
20.7.4 ROESY spectra 580
20.7.5 ROESY and chemical exchange 582
20.7.6 ROESY and TOCSY 583
20.8 Cross-Correlated Relaxation 584
20.8.1 Cross-correlation 584
20.8.2 Cross-correlation of spin interactions 585
20.8.3 Dipole–dipole cross-correlation and angular estimations 586
20.8.4 TROSY 590

Part 8 Appendices 597

Appendix A: Supplementary Material 599

A.1 Euler Angles and Frame Transformations 599
A.1.1 Definition of the Euler angles 599
A.1.2 Euler rotations: first scheme 599
A.1.3 Euler rotations: second scheme 600
A.1.4 Euler rotation matrices 601
A.1.5 Reference-frame orientations 601
A.1.6 Consecutive reference-frame transformations 602
A.1.7 Passive rotations 602
A.1.8 Tensor transformations 603
A.1.9 Intermediate reference frames 604
A.2 Rotations and Cyclic Commutation 604
A.3 Rotation Sandwiches 605
A.4 Spin-1/2 Rotation Operators 606
A.5 Quadrature Detection and Spin Coherences 608
A.6 Secular Approximation 611
A.7 Quadrupolar Interaction 614
A.7.1 Full quadrupolar interaction 614
A.7.2 First-order quadrupolar interaction 614
A.7.3 Higher-order quadrupolar interactions 615
A.8 Strong Coupling 615
A.8.1 Strongly-coupled Spin-1/2 pairs 615
A.8.2 General strongly coupled systems 620
A.9 *J*-Couplings and Magnetic Equivalence 621
A.10 Spin Echo Sandwiches 623
A.10.1 Short-duration limit 625
A.10.2 Long-duration limit 625
A.10.3 Two spin echo sequences 626
A.10.4 Heteronuclear spin echo sequences 627
A.11 Phase Cycling 629
A.11.1 Coherence transfer pathways 629
A.11.2 Coherence transfer amplitudes 630
A.11.3 Coherence orders and phase shifts 631
A.11.4 The pathway phase 632
A.11.5 A sum theorem 633
A.11.6 Pathway selection I 634
A.11.7 Pathway selection II 635
A.11.8 Pathway selection III 637
A.11.9 Selection of a single pathway I 638
A.11.10 Selection of a single pathway II 639
A.11.11 Dual pathway selection 640
A.11.12 Internal phases I 641
A.11.13 Internal phases II 642
A.11.14 Nested phase cycles I 644
A.11.15 Nested phase cycles II 645
A.11.16 Different ways of constructing phase cycles 648

A.12 Coherence Selection by Pulsed Field Gradients 649
A.12.1 Field gradient dephasing 649
A.12.2 Pathway phase 651
A.12.3 Coherence transfer echoes 652
A.12.4 Pathway selection 652
A.12.5 Heteronuclear coherence transfer echoes 652
A.13 Bloch Equations 653
A.14 Chemical Exchange 654
A.14.1 The incoherent dynamics 655
A.14.2 The coherent dynamics 655
A.14.3 The spectrum 656
A.14.4 Longitudinal magnetization exchange 658
A.15 Solomon Equations 660
A.16 Cross-Relaxation Dynamics 662

Appendix B: Symbols and Abbreviations 665

Answers to the Exercises 681

Index 693