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The sources from which some of the examples and exercises have been taken are indicated by the following abbreviations:

- O. Examinations for Mathematical Scholarships and Exhibitions set by Colleges in the University of Oxford;
- C. Examinations for Scholarships and Exhibitions in Mathematics (and, in a few cases, in Natural Science) set by Colleges in the University of Cambridge;
- L.I.C. Examinations for Entrance Scholarships, Exhibitions and Bursaries set by the London Inter-Collegiate Scholarships Board;
- O.C. General Certificate Examination at Scholarship Level set by the Oxford and Cambridge Schools Examination Board;
- L.U. General Certificate Examination at Scholarship Level set by the University of London;
- N.U. General Certificate Examination at Scholarship Level set by the Joint Matriculation Board of the Universities of Manchester, Liverpool, Leeds, Sheffield and Birmingham;
- W. General Certificate Examination at Scholarship Level set by the Welsh Joint Education Committee.