

# CONTENTS

---

## CHAPTER I

### PRELIMINARY REMARKS ON ANALYTICAL GEOMETRY AND VECTOR ANALYSIS

	Page
1. Rectangular Co-ordinates and Vectors . . . . .	1
2. The Area of a Triangle, the Volume of a Tetrahedron, the Vector Multiplication of Vectors . . . . .	12
3. Simple Theorems on Determinants of the Second and Third Order . . . . .	19
4. Affine Transformations and the Multiplication of Determinants .	27

## CHAPTER II

### FUNCTIONS OF SEVERAL VARIABLES AND THEIR DERIVATIVES

1. The Concept of Function in the Case of Several Variables . . .	39
2. Continuity . . . . .	44
3. The Derivatives of a Function . . . . .	50
4. The Total Differential of a Function and its Geometrical Meaning	59
5. Functions of Functions (Compound Functions) and the Intro- duction of New Independent Variables . . . . .	69
6. The Mean Value Theorem and Taylor's Theorem for Functions of Several Variables . . . . .	78
7. The Application of Vector Methods . . . . .	82

## APPENDIX

1. The Principle of the Point of Accumulation in Several Dimen- sions and its Applications . . . . .	95
2. The Concept of Limit for Functions of Several Variables . . . .	101
3. Homogeneous Functions . . . . .	108

## CHAPTER III

DEVELOPMENTS AND APPLICATIONS OF THE  
DIFFERENTIAL CALCULUS

	Page
1. Implicit Functions	111
2. Curves and Surfaces in Implicit Form	122
3. Systems of Functions, Transformations, and Mappings	133
4. Applications	159
5. Families of Curves, Families of Surfaces, and their Envelopes	169
6. Maxima and Minima	183

## APPENDIX

1. Sufficient Conditions for Extreme Values	204
2. Singular Points of Plane Curves	209
3. Singular Points of Surfaces	211
4. Connexion between Euler's and Lagrange's Representations of the Motion of a Fluid	212
5. Tangential Representation of a Closed Curve	213

## CHAPTER IV

## MULTIPLE INTEGRALS

1. Ordinary Integrals as Functions of a Parameter	215
2. The Integral of a Continuous Function over a Region of the Plane or of Space	223
3. Reduction of the Multiple Integral to Repeated Single Integrals	236
4. Transformation of Multiple Integrals	247
5. Improper Integrals	256
6. Geometrical Applications	264
7. Physical Applications	276

## APPENDIX

1. The Existence of the Multiple Integral	287
2. General Formula for the Area (or Volume) of a Region bounded by Segments of Straight Lines or Plane Areas (Guldin's Formula). The Polar Planimeter	294
3. Volumes and Areas in Space of any Number of Dimensions	298
4. Improper Integrals as Functions of a Parameter	307
5. The Fourier Integral	318
6. The Eulerian Integrals (Gamma Function)	323

7. Differentiation and Integration to Fractional Order. Abel's Integral Equation	339
8. Note on the Definition of the Area of a Curved Surface	341

## CHAPTER V

INTEGRATION OVER REGIONS IN SEVERAL  
DIMENSIONS

1. Line Integrals	343
2. Connexion between Line Integrals and Double Integrals in the Plane. (The Integral Theorems of Gauss, Stokes, and Green)	359
3. Interpretation and Applications of the Integral Theorems for the Plane	370
4. Surface Integrals	374
5. Gauss's Theorem and Green's Theorem in Space	384
6. Stokes's Theorem in Space	392
7. The Connexion between Differentiation and Integration for Several Variables	397

## APPENDIX

1. Remarks on Gauss's Theorem and Stokes's Theorem	402
2. Representation of a Source-free Vector Field as a Curl	404

## CHAPTER VI

## DIFFERENTIAL EQUATIONS

1. The Differential Equations of the Motion of a Particle in Three Dimensions	412
2. Examples on the Mechanics of a Particle	418
3. Further Examples of Differential Equations	429
4. Linear Differential Equations	438
5. General Remarks on Differential Equations	450
6. The Potential of Attracting Charges	468
7. Further Examples of Partial Differential Equations	481

## CHAPTER VII

## CALCULUS OF VARIATIONS

1. Introduction	491
2. Euler's Differential Equation in the Simplest Case	497
3. Generalizations	507

## CONTENTS

## CHAPTER VIII

## FUNCTIONS OF A COMPLEX VARIABLE

	Page
1. Introduction	522
2. Foundations of the Theory of Functions of a Complex Variable	530
3. The Integration of Analytic Functions	537
4. Cauchy's Formula and its Applications	545
5. Applications to Complex Integration (Contour Integration)	554
6. Many-valued Functions and Analytic Extension	563

## SUPPLEMENT

Real Numbers and the Concept of Limit	569
Miscellaneous Examples	587
Summary of Important Theorems and Formulae	600
Answers and Hints	623
Index	679