

CONTENTS

| | |
|---|-----|
| INTRODUCTION. REVIEW OF ALGEBRA, ANALYTIC GEOMETRY AND CALCULUS | 1 |
| 0-1 The real number system | 1 |
| 0-2 The complex number system | 2 |
| 0-3 Algebra of real and complex numbers | 4 |
| 0-4 Plane analytic geometry | 8 |
| 0-5 Solid analytic geometry | 9 |
| 0-6 Functions, limits, continuity | 12 |
| 0-7 The elementary transcendental functions | 14 |
| 0-8 The differential calculus | 17 |
| 0-9 The integral calculus | 21 |
| CHAPTER 1. VECTORS | 32 |
| 1-1 Introduction | 32 |
| 1-2 Basic definitions | 33 |
| 1-3 Addition and subtraction of vectors | 34 |
| 1-4 Magnitude of a vector | 36 |
| 1-5 Scalar times vector | 36 |
| 1-6 Applications to geometric theorems | 38 |
| 1-7 Scalar product of two vectors | 40 |
| 1-8 Base vectors | 42 |
| 1-9 Unit vectors, direction cosines, direction numbers | 43 |
| 1-10 Orientation in space | 46 |
| 1-11 The vector product | 47 |
| 1-12 The scalar triple product | 49 |
| 1-13 The vector triple products | 52 |
| 1-14 Vector identities | 53 |
| 1-15 Vector functions of one variable | 54 |
| 1-16 Derivative of a vector function. The velocity vector | 56 |
| 1-17 Properties of the derivative. Higher derivatives | 58 |
| *1-18 Vectors in mechanics | 64 |
| CHAPTER 2. DIFFERENTIAL CALCULUS OF FUNCTIONS OF SEVERAL VARIABLES | 72 |
| 2-1 Functions of several variables | 72 |
| 2-2 Domains and regions | 72 |
| 2-3 Functional notation. Level curves and level surfaces | 74 |
| 2-4 Limits and continuity | 76 |
| 2-5 Partial derivatives | 79 |
| 2-6 Total differential. Fundamental lemma | 82 |
| 2-7 Derivatives and differentials of functions of functions | 86 |
| 2-8 Implicit functions. Inverse functions. Jacobians | 90 |
| 2-9 Geometrical applications | 100 |
| 2-10 The directional derivative | 107 |

| | | |
|--|--|-----|
| 2-11 | Partial derivatives of higher order | 112 |
| 2-12 | Higher derivatives of functions of functions | 114 |
| 2-13 | The Laplacian in polar, cylindrical, and spherical coordinates | 115 |
| 2-14 | Higher derivatives of implicit functions | 117 |
| 2-15 | Maxima and minima of functions of several variables | 120 |
| *2-16 | Maxima and minima for functions with side conditions. Lagrange multipliers | 128 |
| *2-17 | Functional dependence | 132 |
| *2-18 | Derivatives and differences | 136 |
| CHAPTER 3. VECTOR DIFFERENTIAL CALCULUS | | 140 |
| 3-1 | Introduction | 140 |
| 3-2 | Vector fields and scalar fields | 141 |
| 3-3 | The gradient field | 143 |
| 3-4 | The divergence of a vector field | 144 |
| 3-5 | The curl of a vector field | 146 |
| 3-6 | Combined operations | 147 |
| *3-7 | Curvilinear coordinates in space. Orthogonal coordinates | 151 |
| *3-8 | Vector operations in orthogonal curvilinear coordinates | 154 |
| *3-9 | Analytic geometry and vectors in space of more than 3 dimensions | 160 |
| CHAPTER 4. INTEGRAL CALCULUS OF FUNCTIONS OF SEVERAL VARIABLES | | 167 |
| 4-1 | Introduction | 167 |
| 4-2 | Numerical evaluation of definite integrals | 167 |
| 4-3 | Numerical evaluation of indefinite integrals. Elliptic integrals | 176 |
| 4-4 | Improper integrals | 181 |
| *4-5 | Tests for convergence of improper integrals. Numerical evaluation | 185 |
| 4-6 | Double integrals | 191 |
| 4-7 | Triple integrals and multiple integrals in general | 196 |
| 4-8 | Change of variables in integrals | 199 |
| 4-9 | Arc length and surface area | 206 |
| *4-10 | Numerical evaluation of multiple integrals | 211 |
| *4-11 | Improper multiple integrals | 215 |
| 4-12 | Integrals depending on a parameter — Leibnitz's rule | 218 |
| CHAPTER 5. VECTOR INTEGRAL CALCULUS | | 225 |
| PART I. TWO-DIMENSIONAL THEORY | | |
| 5-1 | Introduction | 225 |
| 5-2 | Line integrals in the plane | 227 |
| 5-3 | Integrals with respect to arc length. Basic properties of line integrals | 232 |
| 5-4 | Line integrals as integrals of vectors | 236 |
| 5-5 | Green's theorem | 239 |
| 5-6 | Independence of path. Simply connected domains | 243 |
| 5-7 | Extension of results to multiply connected domains | 252 |
| PART II. THREE-DIMENSIONAL THEORY AND APPLICATIONS | | |
| 5-8 | Line integrals in space | 258 |
| 5-9 | Surfaces in space. Orientability | 260 |

| | | |
|--|--|-----|
| 5-10 | Surface integrals | 262 |
| 5-11 | The divergence theorem | 269 |
| 5-12 | Stokes' theorem | 275 |
| 5-13 | Integrals independent of path. Irrotational fields and solenoidal fields | 279 |
| *5-14 | Change of variables in a multiple integral | 284 |
| *5-15 | Physical applications | 292 |
| CHAPTER 6. INFINITE SERIES | | 302 |
| 6-1 | Introduction | 302 |
| 6-2 | Infinite sequences | 303 |
| 6-3 | Upper and lower limits | 306 |
| 6-4 | Further properties of sequences | 307 |
| 6-5 | Infinite series | 310 |
| 6-6 | Tests for convergence and divergence | 312 |
| 6-7 | Examples of applications of tests for convergence and divergence | 318 |
| *6-8 | Extended ratio test and root test | 324 |
| *6-9 | Computation with series — estimate of error | 326 |
| 6-10 | Operations on series | 332 |
| 6-11 | Sequences and series of functions | 337 |
| 6-12 | Uniform convergence | 338 |
| 6-13 | Weierstrass M -test for uniform convergence | 342 |
| 6-14 | Properties of uniformly convergent series and sequences | 345 |
| 6-15 | Power series | 349 |
| 6-16 | Taylor and Maclaurin series | 354 |
| 6-17 | Taylor's formula with remainder | 357 |
| 6-18 | Further operations on power series | 361 |
| *6-19 | Sequences and series of complex numbers | 364 |
| *6-20 | Sequences and series of functions of several variables | 369 |
| *6-21 | Taylor's formula for functions of several variables | 371 |
| *6-22 | Improper integrals versus infinite series | 372 |
| *6-23 | Improper integrals depending on a parameter — uniform convergence | 378 |
| *6-24 | Laplace transformation. Γ -function and B -function | 380 |
| CHAPTER 7. FOURIER SERIES AND ORTHOGONAL FUNCTIONS | | 387 |
| 7-1 | Trigonometric series | 387 |
| 7-2 | Fourier series | 388 |
| 7-3 | Convergence of Fourier series | 390 |
| 7-4 | Examples — minimizing of square error | 392 |
| 7-5 | Generalizations; Fourier cosine series; Fourier sine series | 399 |
| 7-6 | Remarks on applications of Fourier series | 404 |
| 7-7 | Uniqueness theorem | 406 |
| 7-8 | Proof of fundamental theorem for functions which are continuous, periodic, and piecewise very smooth | 408 |
| 7-9 | Proof of fundamental theorem | 409 |
| 7-10 | Orthogonal functions | 414 |
| *7-11 | Fourier series of orthogonal functions. Completeness | 417 |

| | | |
|--|--|-----|
| *7-12 | Sufficient conditions for completeness | 420 |
| *7-13 | Integration and differentiation of Fourier series | 423 |
| *7-14 | Fourier-Legendre series | 425 |
| *7-15 | Fourier-Bessel series | 429 |
| *7-16 | Orthogonal systems of functions of several variables | 432 |
| *7-17 | Complex form of Fourier series. Fourier integral | 433 |
| CHAPTER 8. ORDINARY DIFFERENTIAL EQUATIONS | | 437 |
| 8-1 | Differential equations | 437 |
| 8-2 | Solutions | 438 |
| 8-3 | The basic problems. Fundamental theorem | 438 |
| 8-4 | Equations of first order and first degree | 441 |
| 8-5 | The general exact equation | 443 |
| 8-6 | Linear equation of first order | 446 |
| 8-7 | Properties of the solution of the linear equation | 450 |
| 8-8 | Graphical and numerical procedure for the first order equation | 455 |
| 8-9 | Linear differential equations of arbitrary order | 458 |
| 8-10 | Linear differential equations with constant coefficients. Homogeneous case | 460 |
| 8-11 | Linear differential equations, nonhomogeneous case | 464 |
| 8-12 | Systems of linear equations with constant coefficients | 468 |
| 8-13 | Applications of linear differential equations | 474 |
| 8-14 | Solution of differential equations by means of Taylor series | 478 |
| CHAPTER 9. FUNCTIONS OF A COMPLEX VARIABLE | | 485 |
| 9-1 | Introduction | 485 |
| 9-2 | The complex number system | 486 |
| 9-3 | Polar form of complex numbers | 488 |
| 9-4 | The exponential function | 490 |
| 9-5 | Sequences and series of complex numbers | 491 |
| 9-6 | Functions of a complex variable | 494 |
| 9-7 | Limits and continuity | 495 |
| 9-8 | Sequences and series of functions | 498 |
| 9-9 | Derivatives and differentials | 501 |
| 9-10 | Integrals | 505 |
| 9-11 | Analytic functions. Cauchy-Riemann equations | 510 |
| 9-12 | Integrals of analytic functions. Cauchy integral theorem | 516 |
| *9-13 | Change of variable in complex integrals | 519 |
| 9-14 | Elementary analytic functions | 521 |
| *9-15 | Inverse functions | 525 |
| 9-16 | The function $\log z$ | 527 |
| 9-17 | The functions a^z , z^a , $\sin^{-1} z$, $\cos^{-1} z$ | 529 |
| 9-18 | Power series as analytic functions | 532 |
| 9-19 | Cauchy's theorem for multiply connected domains | 537 |
| 9-20 | Cauchy's integral formula | 538 |
| 9-21 | Power series expansion of general analytic function | 540 |
| 9-22 | Properties of real and imaginary parts of analytic functions. Poisson integral formula | 545 |

| | | |
|--|---|-----|
| 9-23 | Power series in positive and negative powers — Laurent expansion | 552 |
| 9-24 | Isolated singularities of an analytic function. Zeros and poles | 554 |
| 9-25 | The complex number ∞ | 557 |
| 9-26 | Residues | 562 |
| 9-27 | Residue at infinity | 567 |
| *9-28 | Logarithmic residues — argument principle | 570 |
| 9-29 | Application of residues to evaluation of real integrals | 575 |
| 9-30 | Conformal mapping | 580 |
| 9-31 | Examples of conformal mapping | 582 |
| 9-32 | Applications of conformal mapping. The Dirichlet problem | 591 |
| 9-33 | Dirichlet problem for the half-plane | 592 |
| 9-34 | Conformal mapping in hydrodynamics | 599 |
| 9-35 | Applications of conformal mapping in the theory of elasticity | 602 |
| 9-36 | Further applications of conformal mapping | 603 |
| 9-37 | General formulas for one-to-one mapping. Schwarz-Christoffel transformation | 605 |
| 9-38 | Analytic continuation | 610 |
| 9-39 | Riemann surfaces | 613 |
| CHAPTER 10. PARTIAL DIFFERENTIAL EQUATIONS | | 616 |
| 10-1 | Introduction | 616 |
| 10-2 | Review of equation for forced vibrations of a spring | 617 |
| 10-3 | Case of two particles | 619 |
| 10-4 | Case of N particles | 625 |
| 10-5 | Continuous medium. Fundamental partial differential equation | 631 |
| 10-6 | Classification of partial differential equations. Basic problems | 633 |
| 10-7 | The wave equation in one dimension. Harmonic motion | 635 |
| 10-8 | Properties of solutions of the wave equation | 638 |
| 10-9 | The one-dimensional heat equation. Exponential decay | 642 |
| 10-10 | Properties of solutions of the heat equation | 644 |
| 10-11 | Equilibrium and approach to equilibrium | 645 |
| 10-12 | Forced motion | 647 |
| 10-13 | Equations with variable coefficients. Sturm-Liouville boundary value problems | 652 |
| 10-14 | Equations in two and three dimensions. Separation of variables | 654 |
| 10-15 | Unbounded regions. Continuous spectrum | 657 |
| 10-16 | Numerical methods | 660 |
| 10-17 | Variational methods | 662 |
| 10-18 | Partial differential equations and integral equations | 665 |
| INDEX | | 671 |