

# CONTENTS

	PAGE
SYMBOLS . . . . .	xvii
REFERENCES TO LITERATURE . . . . .	xix
GREEK ALPHABET . . . . .	xx

## CHAPTER I

### PRELIMINARY NOTIONS

1-01. Aerodynamic force . . . . .	1
1-02. Lift and drag . . . . .	2
1-1. Monoplane aircraft . . . . .	3
1-11. Chord of a profile . . . . .	4
1-12. Chord of an aerofoil . . . . .	5
1-13. Aspect ratio . . . . .	5
1-14. Camber . . . . .	6
1-15. Incidence . . . . .	6
1-2. Fluids . . . . .	7
1-21. Velocity . . . . .	8
1-22. Streamlines and paths of particles . . . . .	9
1-23. Stream tubes and filaments . . . . .	10
1-3. Density . . . . .	11
1-4. Pressure . . . . .	11
1-41. Thrust due to pressure . . . . .	12
1-5. The speed of sound . . . . .	13
1-6. Maxwell's definition of viscosity . . . . .	15
1-7. Physical dimensions . . . . .	16
1-71. Aerodynamic force ; dimensional theory . . . . .	17
1-72. Similar systems ; scale effect . . . . .	19
1-73. Coefficients . . . . .	19
1-8. The boundary layer . . . . .	20
1-9. Approximations . . . . .	23
EXAMPLES I . . . . .	26

## CHAPTER II

### BERNOULLI'S THEOREM

2-1. Bernoulli's theorem . . . . .	29
2-11. Incompressible fluid in the gravitational field . . . . .	30
2-12. The constant in Bernoulli's theorem . . . . .	31
2-13. Aerodynamic pressure . . . . .	31
2-2. The Pitot tube . . . . .	32
2-3. The work done by air in expanding . . . . .	33

viii	CONTENTS	
		PAGE
2-31.	Bernoulli's theorem for compressible flow . . . . .	34
2-32.	Application of Bernoulli's theorem to adiabatic expansion . . . . .	34
2-4.	The Venturi tube . . . . .	36
2-41.	Flow of air measured by the Venturi tube . . . . .	37
2-5.	Standard atmosphere . . . . .	37
	EXAMPLES II . . . . .	39

CHAPTER III  
TWO-DIMENSIONAL MOTION

3-0.	Motion in two dimensions . . . . .	42
3-1.	Stream function . . . . .	42
3-11.	Velocity derived from the stream function . . . . .	44
3-12.	Rankine's theorem . . . . .	44
3-13.	The stream function of a uniform wind . . . . .	45
3-14.	Circular cylinder . . . . .	46
3-15.	The dividing streamline . . . . .	47
3-2.	Circulation . . . . .	47
3-21.	Vorticity . . . . .	49
3-22.	Motion of a fluid element . . . . .	50
3-3.	Irrotational motion . . . . .	51
3-31.	Velocity potential . . . . .	52
3-311.	Laplace's equation . . . . .	53
3-32.	Cyclic motion . . . . .	54
3-4.	Complex numbers . . . . .	54
3-41.	The argument . . . . .	55
3-42.	Differentiation . . . . .	57
3-43.	Holomorphic functions . . . . .	57
3-44.	Conjugate functions . . . . .	57
3-45.	The function $\bar{f}(z)$ . . . . .	58
3-47.	The coordinates $z$ and $\bar{z}$ . . . . .	59
3-5.	Cauchy's integral theorem . . . . .	60
3-51.	Singularities . . . . .	60
3-52.	Residues . . . . .	61
3-53.	Cauchy's residue theorem . . . . .	61
3-6.	Conformal mapping . . . . .	62
3-7.	Complex potential . . . . .	63
3-71.	The complex velocity . . . . .	65
3-8.	Application of conformal mapping . . . . .	66
	EXAMPLES III . . . . .	67

CHAPTER IV  
RECTILINEAR VORTICES

4-0.	Two-dimensional vortices . . . . .	70
4-1.	Circular vortex . . . . .	70

	CONTENTS	ix
		PAGE
4-11.	Velocity distribution . . . . .	72
4-12.	Size of a circular vortex . . . . .	73
4-2.	Point rectilinear vortex . . . . .	74
4-21.	Vortex pair . . . . .	74
4-22.	Image of a vortex in a plane . . . . .	75
4-3.	Vortex between parallel planes . . . . .	76
4-4.	Force on a vortex . . . . .	77
4-5.	Mutual action of two vortices . . . . .	77
4-6.	Energy due to a pair of vortices . . . . .	78
4-7.	Continuous line of vortices . . . . .	79
4-71.	Evaluation of the definite integral . . . . .	80
	EXAMPLES IV . . . . .	81

CHAPTER V  
THE CIRCULAR CYLINDER AS AN AEROFOIL

5-0.	Preliminary . . . . .	83
5-1.	The points $z$ and $a^2/z$ . . . . .	83
5-2.	The circle theorem . . . . .	84
5-3.	Circulation about a circular cylinder . . . . .	85
5-31.	Circular cylinder in a wind with circulation . . . . .	86
5-311.	Given stagnation point . . . . .	87
5-32.	The pressure on the cylinder . . . . .	88
5-33.	Force on the cylinder . . . . .	89
5-4.	The theorem of Blasius . . . . .	89
5-41.	Theorem of Blasius in terms of the stream function . . . . .	91
5-5.	The theorem of Kutta and Joukowski . . . . .	91
5-7.	The second circle theorem . . . . .	93
5-72.	Uniform shear flow . . . . .	93
5-74.	Circular cylinder in uniform shear flow . . . . .	94
	EXAMPLES V . . . . .	95

CHAPTER VI  
JOUKOWSKI'S TRANSFORMATION

6-1.	Joukowski's transformation . . . . .	97
6-11.	Circles with centre at the origin . . . . .	98
6-2.	Joukowski fins, rudders, and struts . . . . .	99
6-3.	Circular arc profiles . . . . .	99
6-4.	The general Joukowski profile . . . . .	100
6-5.	Geometrical construction . . . . .	101
6-51.	Mapping a circle . . . . .	102
6-6.	Reversal . . . . .	105
6-7.	Construction of tangents . . . . .	106
6-8.	The airflow . . . . .	106
	EXAMPLES VI . . . . .	108

## CHAPTER VII

## THEORY OF TWO-DIMENSIONAL AEROFOILS

	PAGE
7-0. Types of profile . . . . .	111
7-01. Conditions to be satisfied . . . . .	111
7-02. Origin at the centre of the circle . . . . .	113
7-03. Some properties of profiles obtained by transforming a circle . . . . .	114
7-1. Aerodynamic force . . . . .	114
7-11. Joukowski's hypothesis . . . . .	115
7-12. The lift . . . . .	116
7-13. Lift coefficient . . . . .	116
7-14. Pitching moment coefficient . . . . .	118
7-2. Focus of a profile . . . . .	120
7-21. Metacentric parabola . . . . .	121
7-3. Centre of pressure . . . . .	122
7-31. Centre of pressure of a Joukowski rudder . . . . .	122
7-4. Centroid of the circulation . . . . .	123
7-41. The third axis of a profile . . . . .	124
7-5. Force at a sharp point of a profile . . . . .	125
7-51. The flat aerofoil . . . . .	127
7-6. Kármán-Trefftz profiles . . . . .	128
7-7. Von Mises profiles . . . . .	131
7-8. Carafoli profiles . . . . .	134
EXAMPLES VII . . . . .	134

## CHAPTER VIII

## THIN AEROFOILS

8-0. Geometry of profiles . . . . .	136
8-01. Thin aerofoils of small camber . . . . .	138
8-1. The flat aerofoil . . . . .	138
8-2. The general problem . . . . .	140
8-3. Glauert's method of solution . . . . .	141
8-31. Calculation of the induced velocity . . . . .	141
8-32. Determination of the coefficients $A_n$ . . . . .	141
8-33. The lift . . . . .	142
8-34. The pitching moment . . . . .	144
8-35. Travel of the centre of pressure . . . . .	145
8-36. Unsymmetrical aerofoil with a fixed centre of pressure . . . . .	145
8-37. Effect of operating a flap . . . . .	146
8-4. The pressure method . . . . .	147
8-41. The boundary problem . . . . .	148
8-42. The inverse problem . . . . .	151
8-43. Determination of a camber line . . . . .	151
8-44. Determination of a thickness function . . . . .	154
8-5. Substitution vortex . . . . .	156

8-6. The two-dimensional biplane . . . . .	158
8-7. Approximate theory of the biplane . . . . .	159
8-8. Equal biplane . . . . .	160
EXAMPLES VIII . . . . .	162

## CHAPTER IX

## INDUCED VELOCITY

9-1. Vector notation . . . . .	165
9-2. The equation of motion . . . . .	166
9-21. Kelvin's circulation theorem . . . . .	166
9-3. Vorticity . . . . .	167
9-31. Vortex lines, tubes, and filaments . . . . .	168
9-4. The law of induced velocity . . . . .	169
9-5. Velocity induced by a vortex filament . . . . .	171
9-51. Straight filament . . . . .	171
9-52. The horseshoe vortex . . . . .	172
9-6. Vortex sheet . . . . .	173
9-61. Velocity of a point of a vortex sheet . . . . .	175
9-63. Application to aerofoils . . . . .	176
EXAMPLES IX . . . . .	177

## CHAPTER X

## AEROFOIL OF FINITE ASPECT RATIO

10-1. Steady motion . . . . .	180
10-2. The generation of the vortex system . . . . .	182
10-21. The impulse of a vortex ring . . . . .	184
10-3. The lift . . . . .	185
10-4. Instability of the vortex sheet . . . . .	185
10-41. Tip vortices . . . . .	186
10-5. The velocity of the air . . . . .	186
10-51. The velocity at a point of the aerofoil . . . . .	187
10-6. Aerodynamic force . . . . .	188
10-61. The force $F$ . . . . .	189
10-7. Moments . . . . .	190

## CHAPTER XI

## THE LIFTING LINE THEORY

11-0. Preliminary . . . . .	191
11-01. Geometrical hypotheses . . . . .	191
11-1. Axes of reference . . . . .	191
11-2. Expression for the lift . . . . .	192
11-201. Distribution of load . . . . .	194
11-21. Expression for the induced drag . . . . .	194

	PAGE
11-22. The downwash velocity at the trailing edge - . . . . .	196
11-23. The loading law - . . . . .	197
11-24. Effect of downwash on incidence - . . . . .	197
11-3. The integral equation for the circulation - . . . . .	200
11-4. Problem I; elliptic loading - . . . . .	200
11-41. Elliptic loading; lift and induced drag - . . . . .	202
11-42. Slope of the lift graph for elliptic loading - . . . . .	203
11-43. Change of aspect ratio in elliptic loading - . . . . .	204
11-5. Problem II - . . . . .	204
11-51. The lift - . . . . .	207
11-52. The downwash velocity - . . . . .	207
11-53. The induced drag - . . . . .	208
11-6. Minimum induced drag - . . . . .	209
11-7. The wake far downwind - . . . . .	209
11-8. Lift and drag deduced from the impulse - . . . . .	210
11-9. The rectangular aerofoil - . . . . .	211
11-91. Cylindrical rectangular aerofoil - . . . . .	211
11-92. Method of Betz - . . . . .	211
11-93. Method of Fuchs - . . . . .	213
EXAMPLES XI - . . . . .	213

## CHAPTER XII

## LIFTING SURFACE THEORY

12-1. Velocity induced by a lifting line element - . . . . .	216
12-11. Munk's theorem of stagger - . . . . .	218
12-12. The induced lift - . . . . .	219
12-2. Blenk's method - . . . . .	220
12-21. Rectangular aerofoil - . . . . .	221
12-22. Calculation of the downwash velocity - . . . . .	222
12-23. Side-slipping rectangular aerofoil - . . . . .	224
12-3. Aerofoils of small aspect ratio - . . . . .	225
12-31. The integral equation - . . . . .	226
12-32. Zero aspect ratio - . . . . .	227
12-4. The acceleration potential - . . . . .	228
12-41. Lifting surface - . . . . .	229
12-5. Aerofoil of circular plan - . . . . .	230
12-6. Aerofoil of elliptic plan - . . . . .	231
EXAMPLES XII - . . . . .	232

## CHAPTER XIII

## PROPELLERS

13-1. Propellers - . . . . .	233
13-2. How thrust is developed - . . . . .	234

	PAGE
13-3. The slipstream - . . . . .	235
13-31. Velocity and pressure in the slipstream - . . . . .	237
13-4. Interference velocity - . . . . .	238
13-41. The force on a blade element - . . . . .	239
13-42. Characteristic coefficients - . . . . .	240
13-5. Infinitely many blades - . . . . .	242
13-51. The enclosed propeller - . . . . .	243
13-6. Froude's law - . . . . .	244
13-7. Interference factors - . . . . .	244
EXAMPLES XIII - . . . . .	245

## CHAPTER XIV

## WIND TUNNEL CORRECTIONS

14-0. Preliminary - . . . . .	247
14-1. Nature of the corrections - . . . . .	248
14-2. Boundary conditions - . . . . .	248
14-3. Reduction to two dimensions - . . . . .	249
14-31. Axes of reference - . . . . .	250
14-4. Circular closed section - . . . . .	250
14-41. Elliptic loading - . . . . .	252
14-42. Open jet circular section - . . . . .	253
14-43. Glauert's theorem - . . . . .	253
14-5. Parallel plane boundaries - . . . . .	255
14-6. The general problem - . . . . .	256
14-7. Jacobian elliptic functions - . . . . .	258
14-8. Rectangular section - . . . . .	262
14-9. Mapping an ellipse on a circle - . . . . .	264
14-91. Elliptic section - . . . . .	266
EXAMPLES XIV - . . . . .	267

## CHAPTER XV

## SUBSONIC FLOW

15-0. Preliminary - . . . . .	270
15-01. Thermodynamical considerations - . . . . .	270
15-1. Steady homentropic flow - . . . . .	272
15-2. Irrotational motion - . . . . .	274
15-3. Linear perturbations - . . . . .	275
15-4. Linearised subsonic flow - . . . . .	276
15-41. Distortion of the streamlines - . . . . .	277
15-42. Two-dimensional flow - . . . . .	279
15-43. Lifting line - . . . . .	281
15-44. The hodograph method - . . . . .	281

	PAGE
15-45. The hodograph equations for homentropic flow . . . . .	283
15-46. Velocity correction factor . . . . .	284
EXAMPLES XV . . . . .	286

## CHAPTER XVI

## SUPERSONIC FLOW

16-1. Moving disturbance . . . . .	289
16-15. Thrust due to a supersonic jet . . . . .	290
16-2. Linearised supersonic flow . . . . .	291
16-3. Flow round a corner . . . . .	293
16-32. Characteristics . . . . .	295
16-35. Prandtl-Meyer expansion . . . . .	296
16-36. Complete Prandtl-Meyer expansion . . . . .	298
16-37. Inversion of the deflection equation . . . . .	301
16-4. Shock waves . . . . .	302
16-45. The shock polar . . . . .	304
16-46. Critical angle . . . . .	307
16-6. The flat aerofoil . . . . .	308
16-61. Flat aerofoil with flap . . . . .	310
16-7. Flow past a polygonal profile . . . . .	311
16-71. Flow past a general profile . . . . .	311
EXAMPLES XVI . . . . .	314

## CHAPTER XVII

## SUPERSONIC SWEEPBACK AND DELTA WINGS

17-01. Conical flow . . . . .	317
17-02. Rectangular aerofoil . . . . .	317
17-03. Two-dimensional sweep-back . . . . .	319
17-1. Linearisation for the flat aerofoil . . . . .	320
17-12. Solution of the linearised equation for conical flow . . . . .	322
17-13. The complex variable $\epsilon$ . . . . .	324
17-14. Complex velocity components . . . . .	326
17-15. Compatibility equations . . . . .	326
17-16. Polygonal aerofoils . . . . .	328
17-17. Boundary conditions . . . . .	331
17-18. Circular boundary value problems . . . . .	333
17-19. Non-lifting aerofoils . . . . .	336
17-2. Force on a delta wing . . . . .	338
17-3. Planform $L_0L_0$ . . . . .	340
17-31. Planform $L_iL_i$ . . . . .	342
17-4. Delta wing of constant lift distribution. . . . .	342

	PAGE
17-41. Planform $L_0T_0$ . . . . .	344
17-5. Planform $L_0A$ . . . . .	346
17-51. Planform $L_iA$ . . . . .	349
17-52. Planform $L_0L_i$ . . . . .	350
17-53. Planform $L_0T_i$ . . . . .	354
17-6. Planform $L_iT_i$ . . . . .	356
17-61. Determination of the constant $B$ . . . . .	357
17-62. Planform $L_iL_i$ . . . . .	359
17-63. Determination of the constant $C$ . . . . .	360
17-64. A limit case of planform $L_iL_i$ . . . . .	361
17-65. Planform $L_iA$ . . . . .	362
17-7. Interference of conical flows . . . . .	363
EXAMPLES XVII . . . . .	364

## CHAPTER XVIII

## SIMPLE FLIGHT PROBLEMS

18-0. Preliminary . . . . .	368
18-1. Linear flight . . . . .	369
18-2. Stalling . . . . .	369
18-3. Gliding . . . . .	372
18-31. Straight horizontal flight . . . . .	373
18-32. Sudden increase of incidence . . . . .	374
18-33. Straight side-slip . . . . .	374
18-4. Banked turn . . . . .	375
18-5. Lanchoester's phugoids . . . . .	376
18-51. The phugoid oscillation . . . . .	378
EXAMPLES XVIII . . . . .	378

## CHAPTER XIX

## MOMENTS

19-0. Pitching moment . . . . .	379
19-1. Static stability . . . . .	379
19-11. Metacentric ratio . . . . .	380
19-12. Neutral centre of gravity positions . . . . .	381
19-2. Asymmetric moments . . . . .	383
19-3. The strip hypothesis . . . . .	383
19-4. Moments due to rolling . . . . .	384
19-5. Moments due to yawing . . . . .	385
19-6. Rolling moment due to side-slip . . . . .	386
19-7. Change of axes . . . . .	387
EXAMPLES XIX . . . . .	388

CONTENTS  
CHAPTER XX

STABILITY

	PAGE
20-0. Preliminary . . . . .	389
20-01. Equations of motion . . . . .	389
20-1. Straight flight . . . . .	390
20-11. Simplifying assumption . . . . .	392
20-12. The equations of disturbed horizontal flight . . . . .	393
20-2. The parameter $\mu$ . . . . .	394
20-21. Units . . . . .	394
20-3. Expression in non-dimensional form . . . . .	395
20-4. Dynamical stability . . . . .	397
20-5. Longitudinal stability . . . . .	397
20-51. Routh's discriminant . . . . .	398
20-52. Method of Gates . . . . .	399
20-6. Laplace transform . . . . .	400
20-61. Method of solution . . . . .	401
20-62. Sudden application of elevators . . . . .	402
20-63. Approximate solution of the quartic . . . . .	403
EXAMPLES XX . . . . .	404

CHAPTER XXI

VECTORS

21-1. Scalars and vectors . . . . .	406
21-11. Resolution of vectors . . . . .	407
21-12. The scalar product of two vectors . . . . .	408
21-13. The vector product of two vectors . . . . .	409
21-14. Equations of motion of an aircraft . . . . .	411
21-2. Triple products . . . . .	412
21-3. Vector differentiation; the operator $\nabla$ . . . . .	413
21-31. The acceleration of an air particle . . . . .	415
21-4. Gauss's theorem . . . . .	415
21-5. The equation of continuity . . . . .	417
21-6. The equation of motion . . . . .	418
21-7. Stokes's theorem . . . . .	418
EXAMPLES XXI . . . . .	420
INDEX . . . . .	423