

Contents

Preface	vii
Preface to Fourth Edition	viii
Preface to Third Edition	ix
Preface to Second Edition	xi
Preface to First Edition	xiii
1 Fundamentals	1
1.1 Classical Mechanics	1
1.2 Relativistic Mechanics in Curved Spacetime	10
1.3 Quantum Mechanics	11
1.3.1 Bragg Reflections and Interference	12
1.3.2 Matter Waves	13
1.3.3 Schrödinger Equation	15
1.3.4 Particle Current Conservation	17
1.4 Dirac's Bra-Ket Formalism	18
1.4.1 Basis Transformations	18
1.4.2 Bracket Notation	20
1.4.3 Continuum Limit	22
1.4.4 Generalized Functions	23
1.4.5 Schrödinger Equation in Dirac Notation	25
1.4.6 Momentum States	26
1.4.7 Incompleteness and Poisson's Summation Formula	28
1.5 Observables	31
1.5.1 Uncertainty Relation	32
1.5.2 Density Matrix and Wigner Function	33
1.5.3 Generalization to Many Particles	34
1.6 Time Evolution Operator	34
1.7 Properties of the Time Evolution Operator	37
1.8 Heisenberg Picture of Quantum Mechanics	39
1.9 Interaction Picture and Perturbation Expansion	42
1.10 Time Evolution Amplitude	43
1.11 Fixed-Energy Amplitude	45

1.12	Free-Particle Amplitudes	47
1.13	Quantum Mechanics of General Lagrangian Systems	51
1.14	Particle on the Surface of a Sphere	57
1.15	Spinning Top	59
1.16	Scattering	67
1.16.1	Scattering Matrix	67
1.16.2	Cross Section	68
1.16.3	Born Approximation	70
1.16.4	Partial Wave Expansion and Eikonal Approximation	70
1.16.5	Scattering Amplitude from Time Evolution Amplitude	72
1.16.6	Lippmann-Schwinger Equation	72
1.17	Classical and Quantum Statistics	76
1.17.1	Canonical Ensemble	77
1.17.2	Grand-Canonical Ensemble	77
1.18	Density of States and Tracelog	82
Appendix 1A	Simple Time Evolution Operator	84
Appendix 1B	Convergence of the Fresnel Integral	84
Appendix 1C	The Asymmetric Top	85
Notes and References		87
2	Path Integrals — Elementary Properties and Simple Solutions	89
2.1	Path Integral Representation of Time Evolution Amplitudes	89
2.1.1	Sliced Time Evolution Amplitude	89
2.1.2	Zero-Hamiltonian Path Integral	91
2.1.3	Schrödinger Equation for Time Evolution Amplitude	92
2.1.4	Convergence of the Time-Sliced Evolution Amplitude	93
2.1.5	Time Evolution Amplitude in Momentum Space	94
2.1.6	Quantum-Mechanical Partition Function	96
2.1.7	Feynman's Configuration Space Path Integral	97
2.2	Exact Solution for the Free Particle	101
2.2.1	Direct Solution	101
2.2.2	Fluctuations around the Classical Path	102
2.2.3	Fluctuation Factor	104
2.2.4	Finite Slicing Properties of Free-Particle Amplitude	111
2.3	Exact Solution for Harmonic Oscillator	112
2.3.1	Fluctuations around the Classical Path	112
2.3.2	Fluctuation Factor	114
2.3.3	The $i\eta$ -Prescription and Maslov-Morse Index	115
2.3.4	Continuum Limit	116
2.3.5	Useful Fluctuation Formulas	117
2.3.6	Oscillator Amplitude on Finite Time Lattice	119
2.4	Gelfand-Yaglom Formula	120
2.4.1	Recursive Calculation of Fluctuation Determinant	121
2.4.2	Examples	121

2.4.3	Calculation on Unsliced Time Axis	123
2.4.4	D'Alembert's Construction	124
2.4.5	Another Simple Formula	125
2.4.6	Generalization to D Dimensions	127
2.5	Harmonic Oscillator with Time-Dependent Frequency	127
2.5.1	Coordinate Space	128
2.5.2	Momentum Space	130
2.6	Free-Particle and Oscillator Wave Functions	132
2.7	General Time-Dependent Harmonic Action	134
2.8	Path Integrals and Quantum Statistics	135
2.9	Density Matrix	138
2.10	Quantum Statistics of the Harmonic Oscillator	143
2.11	Time-Dependent Harmonic Potential	148
2.12	Functional Measure in Fourier Space	151
2.13	Classical Limit	154
2.14	Calculation Techniques on Sliced Time Axis via the Poisson Formula	155
2.15	Field-Theoretic Definition of Harmonic Path Integrals by Analytic Regularization	158
2.15.1	Zero-Temperature Evaluation of the Frequency Sum	159
2.15.2	Finite-Temperature Evaluation of the Frequency Sum	162
2.15.3	Quantum-Mechanical Harmonic Oscillator	164
2.15.4	Tracelog of the First-Order Differential Operator	165
2.15.5	Gradient Expansion of the One-Dimensional Tracelog	167
2.15.6	Duality Transformation and Low-Temperature Expansion	168
2.16	Finite- N Behavior of Thermodynamic Quantities	175
2.17	Time Evolution Amplitude of Freely Falling Particle	177
2.18	Charged Particle in Magnetic Field	179
2.18.1	Action	179
2.18.2	Gauge Properties	182
2.18.3	Time-Sliced Path Integration	182
2.18.4	Classical Action	184
2.18.5	Translational Invariance	185
2.19	Charged Particle in Magnetic Field plus Harmonic Potential	186
2.20	Gauge Invariance and Alternative Path Integral Representation	188
2.21	Velocity Path Integral	189
2.22	Path Integral Representation of the Scattering Matrix	190
2.22.1	General Development	190
2.22.2	Improved Formulation	193
2.22.3	Eikonal Approximation to the Scattering Amplitude	194
2.23	Heisenberg Operator Approach to Time Evolution Amplitude	194
2.23.1	Free Particle	195
2.23.2	Harmonic Oscillator	197
2.23.3	Charged Particle in Magnetic Field	197

Appendix 2A	Baker-Campbell-Hausdorff Formula and Magnus Expansion	201
Appendix 2B	Direct Calculation of the Time-Sliced Oscillator Amplitude	204
Appendix 2C	Derivation of Mehler Formula	205
Notes and References	Notes and References	206
3	External Sources, Correlations, and Perturbation Theory	209
3.1	External Sources	209
3.2	Green Function of Harmonic Oscillator	213
3.2.1	Wronski Construction	213
3.2.2	Spectral Representation	217
3.3	Green Functions of First-Order Differential Equation	219
3.3.1	Time-Independent Frequency	219
3.3.2	Time-Dependent Frequency	226
3.4	Summing Spectral Representation of Green Function	229
3.5	Wronski Construction for Periodic and Antiperiodic Green Functions	231
3.6	Time Evolution Amplitude in Presence of Source Term	232
3.7	Time Evolution Amplitude at Fixed Path Average	236
3.8	External Source in Quantum-Statistical Path Integral	237
3.8.1	Continuation of Real-Time Result	238
3.8.2	Calculation at Imaginary Time	242
3.9	Lattice Green Function	249
3.10	Correlation Functions, Generating Functional, and Wick Expansion	249
3.10.1	Real-Time Correlation Functions	252
3.11	Correlation Functions of Charged Particle in Magnetic Field	254
3.12	Correlation Functions in Canonical Path Integral	255
3.12.1	Harmonic Correlation Functions	256
3.12.2	Relations between Various Amplitudes	258
3.12.3	Harmonic Generating Functionals	259
3.13	Particle in Heat Bath	262
3.14	Heat Bath of Photons	266
3.15	Harmonic Oscillator in Ohmic Heat Bath	268
3.16	Harmonic Oscillator in Photon Heat Bath	271
3.17	Perturbation Expansion of Anharmonic Systems	272
3.18	Rayleigh-Schrödinger and Brillouin-Wigner Perturbation Expansion	276
3.19	Level-Shifts and Perturbed Wave Functions from Schrödinger Equation	280
3.20	Calculation of Perturbation Series via Feynman Diagrams	282
3.21	Perturbative Definition of Interacting Path Integrals	287
3.22	Generating Functional of Connected Correlation Functions	288
3.22.1	Connectedness Structure of Correlation Functions	289
3.22.2	Correlation Functions versus Connected Correlation Functions	292

3.22.3	Functional Generation of Vacuum Diagrams	294
3.22.4	Correlation Functions from Vacuum Diagrams	298
3.22.5	Generating Functional for Vertex Functions. Effective Action	300
3.22.6	Ginzburg-Landau Approximation to Generating Functional	305
3.22.7	Composite Fields	306
3.23	Path Integral Calculation of Effective Action by Loop Expansion	307
3.23.1	General Formalism	307
3.23.2	Mean-Field Approximation	308
3.23.3	Corrections from Quadratic Fluctuations	312
3.23.4	Effective Action to Second Order in \hbar	315
3.23.5	Finite-Temperature Two-Loop Effective Action	319
3.23.6	Background Field Method for Effective Action	321
3.24	Nambu-Goldstone Theorem	324
3.25	Effective Classical Potential	326
3.25.1	Effective Classical Boltzmann Factor	327
3.25.2	Effective Classical Hamiltonian	330
3.25.3	High- and Low-Temperature Behavior	331
3.25.4	Alternative Candidate for Effective Classical Potential	332
3.25.5	Harmonic Correlation Function without Zero Mode	333
3.25.6	Perturbation Expansion	334
3.25.7	Effective Potential and Magnetization Curves	336
3.25.8	First-Order Perturbative Result	338
3.26	Perturbative Approach to Scattering Amplitude	340
3.26.1	Generating Functional	340
3.26.2	Application to Scattering Amplitude	341
3.26.3	First Correction to Eikonal Approximation	341
3.26.4	Rayleigh-Schrödinger Expansion of Scattering Amplitude	342
3.27	Functional Determinants from Green Functions	344
Appendix 3A	Matrix Elements for General Potential	350
Appendix 3B	Energy Shifts for $gx^4/4$ -Interaction	351
Appendix 3C	Recursion Relations for Perturbation Coefficients	353
3C.1	One-Dimensional Interaction x^4	353
3C.2	General One-Dimensional Interaction	356
3C.3	Cumulative Treatment of Interactions x^4 and x^3	356
3C.4	Ground-State Energy with External Current	358
3C.5	Recursion Relation for Effective Potential	360
3C.6	Interaction r^4 in D -Dimensional Radial Oscillator	363
3C.7	Interaction r^{2q} in D Dimensions	364
3C.8	Polynomial Interaction in D Dimensions	364
Appendix 3D	Feynman Integrals for $T \neq 0$	364
Notes and References	Notes and References	367

4 Semiclassical Time Evolution Amplitude	369
4.1 Wentzel-Kramers-Brillouin (WKB) Approximation	369
4.2 Saddle Point Approximation	376
4.2.1 Ordinary Integrals	376
4.2.2 Path Integrals	379
4.3 Van Vleck-Pauli-Morette Determinant	385
4.4 Fundamental Composition Law for Semiclassical Time Evolution Amplitude	389
4.5 Semiclassical Fixed-Energy Amplitude	391
4.6 Semiclassical Amplitude in Momentum Space	393
4.7 Semiclassical Quantum-Mechanical Partition Function	395
4.8 Multi-Dimensional Systems	400
4.9 Quantum Corrections to Classical Density of States	405
4.9.1 One-Dimensional Case	406
4.9.2 Arbitrary Dimensions	408
4.9.3 Bilocal Density of States	409
4.9.4 Gradient Expansion of Tracelog of Hamiltonian Operator	411
4.9.5 Local Density of States on Circle	415
4.9.6 Quantum Corrections to Bohr-Sommerfeld Approximation	416
4.10 Thomas-Fermi Model of Neutral Atoms	419
4.10.1 Semiclassical Limit	419
4.10.2 Self-Consistent Field Equation	421
4.10.3 Energy Functional of Thomas-Fermi Atom	423
4.10.4 Calculation of Energies	424
4.10.5 Virial Theorem	427
4.10.6 Exchange Energy	428
4.10.7 Quantum Correction Near Origin	429
4.10.8 Systematic Quantum Corrections to Thomas-Fermi Energies	432
4.11 Classical Action of Coulomb System	436
4.12 Semiclassical Scattering	444
4.12.1 General Formulation	444
4.12.2 Semiclassical Cross Section of Mott Scattering	448
Appendix 4A Semiclassical Quantization for Pure Power Potentials	449
Appendix 4B Derivation of Semiclassical Time Evolution Amplitude	451
Notes and References	455
5 Variational Perturbation Theory	458
5.1 Variational Approach to Effective Classical Partition Function	458
5.2 Local Harmonic Trial Partition Function	459
5.3 Optimal Upper Bound	464
5.4 Accuracy of Variational Approximation	465
5.5 Weakly Bound Ground State Energy in Finite-Range Potential Well	468
5.6 Possible Direct Generalizations	469
5.7 Effective Classical Potential for Anharmonic Oscillator	470

5.8 Particle Densities	475
5.9 Extension to D Dimensions	479
5.10 Application to Coulomb and Yukawa Potentials	481
5.11 Hydrogen Atom in Strong Magnetic Field	484
5.11.1 Weak-Field Behavior	488
5.11.2 Effective Classical Hamiltonian	488
5.12 Variational Approach to Excitation Energies	492
5.13 Systematic Improvement of Feynman-Kleinert Approximation	496
5.14 Applications of Variational Perturbation Expansion	498
5.14.1 Anharmonic Oscillator at $T = 0$	499
5.14.2 Anharmonic Oscillator for $T > 0$	501
5.15 Convergence of Variational Perturbation Expansion	505
5.16 Variational Perturbation Theory for Strong-Coupling Expansion	512
5.17 General Strong-Coupling Expansions	515
5.18 Variational Interpolation between Weak and Strong-Coupling Expansions	518
5.19 Systematic Improvement of Excited Energies	520
5.20 Variational Treatment of Double-Well Potential	521
5.21 Higher-Order Effective Classical Potential for Nonpolynomial Interactions	523
5.21.1 Evaluation of Path Integrals	524
5.21.2 Higher-Order Smearing Formula in D Dimensions	525
5.21.3 Isotropic Second-Order Approximation to Coulomb Problem	527
5.21.4 Anisotropic Second-Order Approximation to Coulomb Problem	528
5.21.5 Zero-Temperature Limit	529
5.22 Polaron	533
5.22.1 Partition Function	535
5.22.2 Harmonic Trial System	537
5.22.3 Effective Mass	542
5.22.4 Second-Order Correction	543
5.22.5 Polaron in Magnetic Field, Bipolarons, etc.	544
5.22.6 Variational Interpolation for Polaron Energy and Mass	545
5.23 Density Matrices	548
5.23.1 Harmonic Oscillator	548
5.23.2 Variational Perturbation Theory for Density Matrices	550
5.23.3 Smearing Formula for Density Matrices	552
5.23.4 First-Order Variational Approximation	554
5.23.5 Smearing Formula in Higher Spatial Dimensions	558
Appendix 5A Feynman Integrals for $T \neq 0$ without Zero Frequency	560
Appendix 5B Proof of Scaling Relation for the Extrema of W_N	562
Appendix 5C Second-Order Shift of Polaron Energy	564
Notes and References	565

6 Path Integrals with Topological Constraints	571
6.1 Point Particle on Circle	571
6.2 Infinite Wall	575
6.3 Point Particle in Box	579
6.4 Strong-Coupling Theory for Particle in Box	582
6.4.1 Partition Function	583
6.4.2 Perturbation Expansion	583
6.4.3 Variational Strong-Coupling Approximations	585
6.4.4 Special Properties of Expansion	587
6.4.5 Exponentially Fast Convergence	588
Notes and References	589
7 Many Particle Orbits — Statistics and Second Quantization	591
7.1 Ensembles of Bose and Fermi Particle Orbits	592
7.2 Bose-Einstein Condensation	599
7.2.1 Free Bose Gas	599
7.2.2 Bose Gas in Finite Box	607
7.2.3 Effect of Interactions	609
7.2.4 Bose-Einstein Condensation in Harmonic Trap	615
7.2.5 Thermodynamic Functions	615
7.2.6 Critical Temperature	617
7.2.7 More General Anisotropic Trap	620
7.2.8 Rotating Bose-Einstein Gas	621
7.2.9 Finite-Size Corrections	622
7.2.10 Entropy and Specific Heat	623
7.2.11 Interactions in Harmonic Trap	626
7.3 Gas of Free Fermions	630
7.4 Statistics Interaction	635
7.5 Fractional Statistics	640
7.6 Second-Quantized Bose Fields	641
7.7 Fluctuating Bose Fields	644
7.8 Coherent States	650
7.9 Second-Quantized Fermi Fields	654
7.10 Fluctuating Fermi Fields	654
7.10.1 Grassmann Variables	654
7.10.2 Fermionic Functional Determinant	657
7.10.3 Coherent States for Fermions	661
7.11 Hilbert Space of Quantized Grassmann Variable	663
7.11.1 Single Real Grassmann Variable	663
7.11.2 Quantizing Harmonic Oscillator with Grassmann Variables	666
7.11.3 Spin System with Grassmann Variables	667
7.12 External Sources in a^*, a -Path Integral	672
7.13 Generalization to Pair Terms	674
7.14 Spatial Degrees of Freedom	676

7.14.1 Grand-Canonical Ensemble of Particle Orbits from Free Fluctuating Field	676
7.14.2 First versus Second Quantization	678
7.14.3 Interacting Fields	678
7.14.4 Effective Classical Field Theory	679
7.15 Bosonization	681
7.15.1 Collective Field	682
7.15.2 Bosonized versus Original Theory	684
Appendix 7A Treatment of Singularities in Zeta-Function	686
7A.1 Finite Box	687
7A.2 Harmonic Trap	689
Appendix 7B Experimental versus Theoretical Would-be Critical Temperature	691
Notes and References	692
8 Path Integrals in Polar and Spherical Coordinates	697
8.1 Angular Decomposition in Two Dimensions	697
8.2 Trouble with Feynman's Path Integral Formula in Radial Coordinates	700
8.3 Cautionary Remarks	704
8.4 Time Slicing Corrections	707
8.5 Angular Decomposition in Three and More Dimensions	711
8.5.1 Three Dimensions	712
8.5.2 D Dimensions	714
8.6 Radial Path Integral for Harmonic Oscillator and Free Particle	720
8.7 Particle near the Surface of a Sphere in D Dimensions	721
8.8 Angular Barriers near the Surface of a Sphere	724
8.8.1 Angular Barriers in Three Dimensions	725
8.8.2 Angular Barriers in Four Dimensions	730
8.9 Motion on a Sphere in D Dimensions	734
8.10 Path Integrals on Group Spaces	739
8.11 Path Integral of Spinning Top	741
8.12 Path Integral of Spinning Particle	743
8.13 Berry Phase	748
8.14 Spin Precession	748
Notes and References	750
9 Wave Functions	752
9.1 Free Particle in D Dimensions	752
9.2 Harmonic Oscillator in D Dimensions	755
9.3 Free Particle from $\omega \rightarrow 0$ -Limit of Oscillator	761
9.4 Charged Particle in Uniform Magnetic Field	763
9.5 Dirac δ -Function Potential	770
Notes and References	772

10 Spaces with Curvature and Torsion	773
10.1 Einstein's Equivalence Principle	774
10.2 Classical Motion of Mass Point in General Metric-Affine Space	775
10.2.1 Equations of Motion	775
10.2.2 Nonholonomic Mapping to Spaces with Torsion	778
10.2.3 New Equivalence Principle	784
10.2.4 Classical Action Principle for Spaces with Curvature and Torsion	784
10.3 Path Integral in Metric-Affine Space	789
10.3.1 Nonholonomic Transformation of Action	789
10.3.2 Measure of Path Integration	794
10.4 Completing the Solution of Path Integral on Surface of Sphere .	800
10.5 External Potentials and Vector Potentials	802
10.6 Perturbative Calculation of Path Integrals in Curved Space	804
10.6.1 Free and Interacting Parts of Action	804
10.6.2 Zero Temperature	807
10.7 Model Study of Coordinate Invariance	809
10.7.1 Diagrammatic Expansion	811
10.7.2 Diagrammatic Expansion in d Time Dimensions	813
10.8 Calculating Loop Diagrams	814
10.8.1 Reformulation in Configuration Space	821
10.8.2 Integrals over Products of Two Distributions	822
10.8.3 Integrals over Products of Four Distributions	823
10.9 Distributions as Limits of Bessel Function	825
10.9.1 Correlation Function and Derivatives	825
10.9.2 Integrals over Products of Two Distributions	827
10.9.3 Integrals over Products of Four Distributions	828
10.10 Simple Rules for Calculating Singular Integrals	830
10.11 Perturbative Calculation on Finite Time Intervals	835
10.11.1 Diagrammatic Elements	836
10.11.2 Cumulant Expansion of D -Dimensional Free-Particle Amplitude in Curvilinear Coordinates	837
10.11.3 Propagator in $1 - \epsilon$ Time Dimensions	839
10.11.4 Coordinate Independence for Dirichlet Boundary Conditions	840
10.11.5 Time Evolution Amplitude in Curved Space	846
10.11.6 Covariant Results for Arbitrary Coordinates	852
10.12 Effective Classical Potential in Curved Space	857
10.12.1 Covariant Fluctuation Expansion	858
10.12.2 Arbitrariness of q_0^μ	861
10.12.3 Zero-Mode Properties	862
10.12.4 Covariant Perturbation Expansion	865
10.12.5 Covariant Result from Noncovariant Expansion	866
10.12.6 Particle on Unit Sphere	869

10.13 Covariant Effective Action for Quantum Particle with Coordinate-Dependent Mass	871
10.13.1 Formulating the Problem	872
10.13.2 Gradient Expansion	875
Appendix 10A Nonholonomic Gauge Transformations in Electromagnetism	875
10A.1 Gradient Representation of Magnetic Field of Current Loops	876
10A.2 Generating Magnetic Fields by Multivalued Gauge Transformations	880
10A.3 Magnetic Monopoles	881
10A.4 Minimal Magnetic Coupling of Particles from Multivalued Gauge Transformations	883
10A.5 Gauge Field Representation of Current Loops and Monopoles	884
Appendix 10B Comparison of Multivalued Basis Tetrads with Vierbein Fields	886
Appendix 10C Cancellation of Powers of $\delta(0)$	888
Notes and References	890
11 Schrödinger Equation in General Metric-Affine Spaces	894
11.1 Integral Equation for Time Evolution Amplitude	894
11.1.1 From Recursion Relation to Schrödinger Equation	895
11.1.2 Alternative Evaluation	898
11.2 Equivalent Path Integral Representations	901
11.3 Potentials and Vector Potentials	905
11.4 Unitarity Problem	906
11.5 Alternative Attempts	909
11.6 DeWitt-Seeley Expansion of Time Evolution Amplitude	910
Appendix 11A Cancellations in Effective Potential	914
Appendix 11B DeWitt's Amplitude	916
Notes and References	917
12 New Path Integral Formula for Singular Potentials	918
12.1 Path Collapse in Feynman's formula for the Coulomb System	918
12.2 Stable Path Integral with Singular Potentials	921
12.3 Time-Dependent Regularization	926
12.4 Relation to Schrödinger Theory. Wave Functions	928
Notes and References	930
13 Path Integral of Coulomb System	931
13.1 Pseudotime Evolution Amplitude	931
13.2 Solution for the Two-Dimensional Coulomb System	933
13.3 Absence of Time Slicing Corrections for $D = 2$	938
13.4 Solution for the Three-Dimensional Coulomb System	943
13.5 Absence of Time Slicing Corrections for $D = 3$	949

13.6	Geometric Argument for Absence of Time Slicing Corrections	951
13.7	Comparison with Schrödinger Theory	952
13.8	Angular Decomposition of Amplitude, and Radial Wave Functions	957
13.9	Remarks on Geometry of Four-Dimensional u^μ -Space	961
13.10	Runge-Lenz-Pauli Group of Degeneracy	963
13.11	Solution in Momentum Space	964
13.11.1	Another Form of Action	968
Appendix 13A	Dynamical Group of Coulomb States	969
Notes and References	972
14	Solution of Further Path Integrals by Duru-Kleinert Method	974
14.1	One-Dimensional Systems	974
14.2	Derivation of the Effective Potential	978
14.3	Comparison with Schrödinger Quantum Mechanics	982
14.4	Applications	983
14.4.1	Radial Harmonic Oscillator and Morse System	983
14.4.2	Radial Coulomb System and Morse System	985
14.4.3	Equivalence of Radial Coulomb System and Radial Oscillator	987
14.4.4	Angular Barrier near Sphere, and Rosen-Morse Potential	994
14.4.5	Angular Barrier near Four-Dimensional Sphere, and General Rosen-Morse Potential	997
14.4.6	Hulthén Potential and General Rosen-Morse Potential	1000
14.4.7	Extended Hulthén Potential and General Rosen-Morse Potential	1002
14.5	D -Dimensional Systems	1003
14.6	Path Integral of the Dionium Atom	1004
14.6.1	Formal Solution	1005
14.6.2	Absence of Time Slicing Corrections	1009
14.7	Time-Dependent Duru-Kleinert Transformation	1012
Appendix 14A	Affine Connection of Dionium Atom	1015
Appendix 14B	Algebraic Aspects of Dionium States	1016
Notes and References	1016
15	Path Integrals in Polymer Physics	1019
15.1	Polymers and Ideal Random Chains	1019
15.2	Moments of End-to-End Distribution	1021
15.3	Exact End-to-End Distribution in Three Dimensions	1024
15.4	Short-Distance Expansion for Long Polymer	1026
15.5	Saddle Point Approximation to Three-Dimensional End-to-End Distribution	1028
15.6	Path Integral for Continuous Gaussian Distribution	1029
15.7	Stiff Polymers	1032
15.7.1	Sliced Path Integral	1034

15.7.2	Relation to Classical Heisenberg Model	1035
15.7.3	End-to-End Distribution	1037
15.7.4	Moments of End-to-End Distribution	1037
15.8	Continuum Formulation	1038
15.8.1	Path Integral	1038
15.8.2	Correlation Functions and Moments	1039
15.9	Schrödinger Equation and Recursive Solution for Moments	1043
15.9.1	Setting up the Schrödinger Equation	1043
15.9.2	Recursive Solution of Schrödinger Equation	1044
15.9.3	From Moments to End-to-End Distribution for $D = 3$	1047
15.9.4	Large-Stiffness Approximation to End-to-End Distribution	1049
15.9.5	Higher Loop Corrections	1054
15.10	Excluded-Volume Effects	1062
15.11	Flory's Argument	1069
15.12	Polymer Field Theory	1070
15.13	Fermi Fields for Self-Avoiding Lines	1077
Appendix 15A	Basic Integrals	1078
Appendix 15B	Loop Integrals	1079
Appendix 15C	Integrals Involving Modified Green Function	1080
Notes and References	1081
16	Polymers and Particle Orbits in Multiply Connected Spaces	1084
16.1	Simple Model for Entangled Polymers	1084
16.2	Entangled Fluctuating Particle Orbit: Aharonov-Bohm Effect	1088
16.3	Aharonov-Bohm Effect and Fractional Statistics	1096
16.4	Self-Entanglement of Polymer	1101
16.5	The Gauss Invariant of Two Curves	1115
16.6	Bound States of Polymers and Ribbons	1117
16.7	Chern-Simons Theory of Entanglements	1124
16.8	Entangled Pair of Polymers	1127
16.8.1	Polymer Field Theory for Probabilities	1129
16.8.2	Calculation of Partition Function	1130
16.8.3	Calculation of Numerator in Second Moment	1132
16.8.4	First Diagram in Fig. 16.23	1134
16.8.5	Second and Third Diagrams in Fig. 16.23	1135
16.8.6	Fourth Diagram in Fig. 16.23	1136
16.8.7	Second Topological Moment	1137
16.9	Chern-Simons Theory of Statistical Interaction	1137
16.10	Second-Quantized Anyon Fields	1140
16.11	Fractional Quantum Hall Effect	1143
16.12	Anyonic Superconductivity	1147
16.13	Non-Abelian Chern-Simons Theory	1149
Appendix 16A	Calculation of Feynman Diagrams in Polymer Entanglement	1151

Appendix 16B	Kauffman and BLM/Ho polynomials	1153
Appendix 16C	Skein Relation between Wilson Loop Integrals	1153
Appendix 16D	London Equations	1156
Appendix 16E	Hall Effect in Electron Gas	1158
Notes and References	1158
7 Tunneling		1164
17.1	Double-Well Potential	1164
17.2	Classical Solutions — Kinks and Antikinks	1167
17.3	Quadratic Fluctuations	1171
17.3.1	Zero-Eigenvalue Mode	1177
17.3.2	Continuum Part of Fluctuation Factor	1181
17.4	General Formula for Eigenvalue Ratios	1183
17.5	Fluctuation Determinant from Classical Solution	1185
17.6	Wave Functions of Double-Well	1189
17.7	Gas of Kinks and Antikinks and Level Splitting Formula	1190
17.8	Fluctuation Correction to Level Splitting	1194
17.9	Tunneling and Decay	1199
17.10	Large-Order Behavior of Perturbation Expansions	1207
17.10.1	Growth Properties of Expansion Coefficients	1208
17.10.2	Semiclassical Large-Order Behavior	1211
17.10.3	Fluctuation Correction to the Imaginary Part and Large-Order Behavior	1216
17.10.4	Variational Approach to Tunneling. Perturbation Coefficients to All Orders	1219
17.10.5	Convergence of Variational Perturbation Expansion	1227
17.11	Decay of Supercurrent in Thin Closed Wire	1235
17.12	Decay of Metastable Thermodynamic Phases	1247
17.13	Decay of Metastable Vacuum State in Quantum Field Theory	1254
17.14	Crossover from Quantum Tunneling to Thermally Driven Decay	1255
Appendix 17A	Feynman Integrals for Fluctuation Correction	1257
Notes and References	1259
8 Nonequilibrium Quantum Statistics		1262
18.1	Linear Response and Time-Dependent Green Functions for $T \neq 0$	1262
18.2	Spectral Representations of Green Functions for $T \neq 0$	1265
18.3	Other Important Green Functions	1268
18.4	Hermitian Adjoint Operators	1271
18.5	Harmonic Oscillator Green Functions for $T \neq 0$	1272
18.5.1	Creation Annihilation Operators	1272
18.5.2	Real Field Operators	1275
18.6	Nonequilibrium Green Functions	1277
18.7	Perturbation Theory for Nonequilibrium Green Functions	1286
18.8	Path Integral Coupled to Thermal Reservoir	1289

18.9	Fokker-Planck Equation	1295
18.9.1	Canonical Path Integral for Probability Distribution	1296
18.9.2	Solving the Operator Ordering Problem	1298
18.9.3	Strong Damping	1303
18.10	Langevin Equations	1307
18.11	Path Integral Solution of Klein-Kramers Equation	1311
18.12	Stochastic Quantization	1312
18.13	Stochastic Calculus	1316
18.13.1	Kubo's stochastic Liouville equation	1316
18.13.2	From Kubo's to Fokker-Planck Equations	1317
18.13.3	Itô's Lemma	1320
18.14	Solving the Langevin Equation	1323
18.15	Heisenberg Picture for Probability Evolution	1327
18.16	Supersymmetry	1330
18.17	Stochastic Quantum Liouville Equation	1332
18.18	Master Equation for Time Evolution	1334
18.19	Relation to Quantum Langevin Equation	1336
18.20	Electromagnetic Dissipation and Decoherence	1337
18.20.1	Forward–Backward Path Integral	1337
18.20.2	Master Equation for Time Evolution in Photon Bath	1340
18.20.3	Line Width	1341
18.20.4	Lamb shift	1342
18.20.5	Langevin Equations	1346
18.21	Fokker-Planck Equation in Spaces with Curvature and Torsion	1347
18.22	Stochastic Interpretation of Quantum-Mechanical Amplitudes	1348
18.23	Stochastic Equation for Schrödinger Wave Function	1350
18.24	Real Stochastic and Deterministic Equation for Schrödinger Wave Function	1351
18.24.1	Stochastic Differential Equation	1352
18.24.2	Equation for Noise Average	1352
18.24.3	Harmonic Oscillator	1353
18.24.4	General Potential	1353
18.24.5	Deterministic Equation	1354
Appendix 18A	Inequalities for Diagonal Green Functions	1355
Appendix 18B	General Generating Functional	1359
Appendix 18C	Wick Decomposition of Operator Products	1363
Notes and References	1364
19 Relativistic Particle Orbits		1368
19.1	Special Features of Relativistic Path Integrals	1370
19.1.1	Simplest Gauge Fixing	1373
19.1.2	Partition Function of Ensemble of Closed Particle Loops	1375
19.1.3	Fixed-Energy Amplitude	1376
19.2	Tunneling in Relativistic Physics	1377

19.2.1	Decay Rate of Vacuum in Electric Field	1377
19.2.2	Birth of Universe	1386
19.2.3	Friedmann Model	1392
19.2.4	Tunneling of Expanding Universe	1396
19.3	Relativistic Coulomb System	1397
19.4	Relativistic Particle in Electromagnetic Field	1400
19.4.1	Action and Partition Function	1401
19.4.2	Perturbation Expansion	1401
19.4.3	Lowest-Order Vacuum Polarization	1404
19.5	Path Integral for Spin-1/2 Particle	1408
19.5.1	Dirac Theory	1408
19.5.2	Path Integral	1412
19.5.3	Amplitude with Electromagnetic Interaction	1414
19.5.4	Effective Action in Electromagnetic Field	1417
19.5.5	Perturbation Expansion	1418
19.5.6	Vacuum Polarization	1419
19.6	Supersymmetry	1421
19.6.1	Global Invariance	1421
19.6.2	Local Invariance	1422
Appendix 19A	Proof of Same Quantum Physics of Modified Action . .	1424
Notes and References	1426
20	Path Integrals and Financial Markets	1428
20.1	Fluctuation Properties of Financial Assets	1428
20.1.1	Harmonic Approximation to Fluctuations	1430
20.1.2	Lévy Distributions	1432
20.1.3	Truncated Lévy Distributions	1434
20.1.4	Asymmetric Truncated Lévy Distributions	1439
20.1.5	Gamma Distribution	1442
20.1.6	Boltzmann Distribution	1443
20.1.7	Student or Tsallis Distribution	1446
20.1.8	Tsallis Distribution in Momentum Space	1448
20.1.9	Relativistic Particle Boltzmann Distribution	1449
20.1.10	Meixner Distributions	1450
20.1.11	Generalized Hyperbolic Distributions	1451
20.1.12	Debye-Waller Factor for Non-Gaussian Fluctuations . .	1454
20.1.13	Path Integral for Non-Gaussian Distribution	1454
20.1.14	Time Evolution of Distribution	1457
20.1.15	Central Limit Theorem	1457
20.1.16	Additivity Property of Noises and Hamiltonians	1459
20.1.17	Lévy-Khintchine Formula	1460
20.1.18	Semigroup Property of Asset Distributions	1461
20.1.19	Time Evolution of Moments of Distribution	1463
20.1.20	Boltzmann Distribution	1464

20.1.21	Fourier-Transformed Tsallis Distribution	1467
20.1.22	Superposition of Gaussian Distributions	1468
20.1.23	Fokker-Planck-Type Equation	1470
20.1.24	Kramers-Moyal Equation	1471
20.2	Itô-like Formula for Non-Gaussian Distributions	1473
20.2.1	Continuous Time	1473
20.2.2	Discrete Times	1476
20.3	Martingales	1477
20.3.1	Gaussian Martingales	1477
20.3.2	Non-Gaussian Martingale Distributions	1479
20.4	Origin of Semi-Heavy Tails	1481
20.4.1	Pair of Stochastic Differential Equations	1482
20.4.2	Fokker-Planck Equation	1482
20.4.3	Solution of Fokker-Planck Equation	1485
20.4.4	Pure x -Distribution	1487
20.4.5	Long-Time Behavior	1488
20.4.6	Tail Behavior for all Times	1492
20.4.7	Path Integral Calculation	1494
20.4.8	Natural Martingale Distribution	1495
20.5	Time Series	1496
20.6	Spectral Decomposition of Power Behaviors	1497
20.7	Option Pricing	1498
20.7.1	Black-Scholes Option Pricing Model	1499
20.7.2	Evolution Equations of Portfolios with Options	1501
20.7.3	Option Pricing for Gaussian Fluctuations	1505
20.7.4	Option Pricing for Boltzmann Distribution	1509
20.7.5	Option Pricing for General Non-Gaussian Fluctuations .	1509
20.7.6	Option Pricing for Fluctuating Variance	1512
20.7.7	Perturbation Expansion and Smile	1514
Appendix 20A	Large- x Behavior of Truncated Lévy Distribution . .	1517
Appendix 20B	Gaussian Weight	1520
Appendix 20C	Comparison with Dow-Jones Data	1521
Notes and References	1522
Index		1529