

CONTENTS

SECTION I Fundamental Concepts

CHAPTER 1. NUMBERS AND POINTS

	PAGE
§ 1. Prerequisites	1
§ 2. The Plane and Sphere of Complex Numbers	2
§ 3. Point Sets and Sets of Numbers	5
§ 4. Paths, Regions, Continua	13

CHAPTER 2. FUNCTIONS OF A COMPLEX VARIABLE

§ 5. The Concept of a Most General (Single-valued) Function of a Complex Variable	21
§ 6. Continuity and Differentiability	23
§ 7. The Cauchy-Riemann Differential Equations	28

SECTION II Integral Theorems

CHAPTER 3. THE INTEGRAL OF A CONTINUOUS FUNCTION

§ 8. Definition of the Definite Integral	32
§ 9. Existence Theorem for the Definite Integral	34
§ 10. Evaluation of Definite Integrals	38
§ 11. Elementary Integral Theorems	44

CHAPTER 4. CAUCHY'S INTEGRAL THEOREM	PAGE
§ 12. Formulation of the Theorem	47
§ 13. Proof of the Fundamental Theorem	49
§ 14. Simple Consequences and Extensions . . .	55
CHAPTER 5. CAUCHY'S INTEGRAL FORMULAS	
§ 15. The Fundamental Formula	61
§ 16. Integral Formulas for the Derivatives . .	62
 SECTION III	
Series and the Expansion of Analytic Functions in Series	
CHAPTER 6. SERIES WITH VARIABLE TERMS	
§ 17. Domain of Convergence	67
§ 18. Uniform Convergence	71
§ 19. Uniformly Convergent Series of Analytic Functions	73
CHAPTER 7. THE EXPANSION OF ANALYTIC FUNCTIONS IN POWER SERIES	
§ 20. Expansion and Identity Theorems for Power Series	79
§ 21. The Identity Theorem for Analytic Func- tions	85
CHAPTER 8. ANALYTIC CONTINUATION AND COMPLETE DEFINITION OF ANALYTIC FUNCTIONS	
§ 22. The Principle of Analytic Continuation .	92
§ 23. The Elementary Functions	96
§ 24. Continuation by Means of Power Series and Complete Definition of Analytic Functions	98
§ 25. The Monodromy Theorem	105
§ 26. Examples of Multiple-valued Functions .	107

CONTENTS

vii

PAGE

CHAPTER 9. ENTIRE TRANSCENDENTAL FUNCTIONS

- | | |
|--|-----|
| § 27. Definitions | 112 |
| § 28. Behavior for Large $ z $ | 112 |

**SECTION IV
Singularities****CHAPTER 10. THE LAURENT EXPANSION**

- | | |
|--------------------------------------|-----|
| § 29. The Expansion | 117 |
| § 30. Remarks and Examples | 119 |

CHAPTER 11. THE VARIOUS TYPES OF SINGULARITIES

- | | |
|--|-----|
| § 31. Essential and Non-essential Singularities or Poles | 123 |
| § 32. Behavior of Analytic Functions at Infinity | 126 |
| § 33. The Residue Theorem | 129 |
| § 34. Inverses of Analytic Functions | 134 |
| § 35. Rational Functions | 137 |

BIBLIOGRAPHY 140**INDEX** 143