

TABLE OF CONTENTS

PREFACE TO THE FIRST EDITION	vii
PREFACE TO THE SECOND EDITION	ix
ABBREVIATED REFERENCES	x
PREFACE TO THE THIRD EDITION	xi

I. STOCHASTIC VARIABLES

1. Definition	1
2. Averages	5
3. Multivariate distributions	10
4. Addition of stochastic variables	14
5. Transformation of variables	17
6. The Gaussian distribution	23
7. The central limit theorem	26

II. RANDOM EVENTS

1. Definition	30
2. The Poisson distribution	33
3. Alternative description of random events	35
4. The inverse formula	40
5. The correlation functions	41
6. Waiting times	44
7. Factorial correlation functions	47

III. STOCHASTIC PROCESSES

1. Definition	52
2. Stochastic processes in physics	55
3. Fourier transformation of stationary processes	58
4. The hierarchy of distribution functions	61
5. The vibrating string and random fields	64
6. Branching processes	69

IV. MARKOV PROCESSES

1. The Markov property	73
2. The Chapman–Kolmogorov equation	78
3. Stationary Markov processes	81
4. The extraction of a subensemble	86
5. Markov chains	89
6. The decay process	93

V. THE MASTER EQUATION	
1. Derivation	96
2. The class of \mathbb{W} -matrices	100
3. The long-time limit	104
4. Closed, isolated, physical systems	108
5. The increase of entropy	111
6. Proof of detailed balance	114
7. Expansion in eigenfunctions	117
8. The macroscopic equation	122
9. The adjoint equation	127
10. Other equations related to the master equation	129
VI. ONE-STEP PROCESSES	
1. Definition; the Poisson process	134
2. Random walk with continuous time	136
3. General properties of one-step processes	139
4. Examples of linear one-step processes	143
5. Natural boundaries	147
6. Solution of linear one-step processes with natural boundaries	149
7. Artificial boundaries	153
8. Artificial boundaries and normal modes	157
9. Nonlinear one-step processes	161
VII. CHEMICAL REACTIONS	
1. Kinematics of chemical reactions	166
2. Dynamics of chemical reactions	171
3. The stationary solution	173
4. Open systems	176
5. Unimolecular reactions	178
6. Collective systems	182
7. Composite Markov processes	186
VIII. THE FOKKER-PLANCK EQUATION	
1. Introduction	193
2. Derivation of the Fokker-Planck equation	197
3. Brownian motion	200
4. The Rayleigh particle	204
5. Application to one-step processes	207
6. The multivariate Fokker-Planck equation	210
7. Kramers' equation	215
IX. THE LANGEVIN APPROACH	
1. Langevin treatment of Brownian motion	219
2. Applications	221

3. Relation to Fokker-Planck equation	224
4. The Langevin approach	227
5. Discussion of the Itô-Stratonovich dilemma	232
6. Non-Gaussian white noise	237
7. Colored noise	240
X. THE EXPANSION OF THE MASTER EQUATION	
1. Introduction to the expansion	244
2. General formulation of the expansion method	248
3. The emergence of the macroscopic law	254
4. The linear noise approximation	258
5. Expansion of a multivariate master equation	263
6. Higher orders	267
XI. THE DIFFUSION TYPE	
1. Master equations of diffusion type	273
2. Diffusion in an external field	276
3. Diffusion in an inhomogeneous medium	279
4. Multivariate diffusion equation	282
5. The limit of zero fluctuations	287
XII. FIRST-PASSAGE PROBLEMS	
1. The absorbing boundary approach	292
2. The approach through the adjoint equation - Discrete case	298
3. The approach through the adjoint equation - Continuous case	303
4. The renewal approach	307
5. Boundaries of the Smoluchowski equation	312
6. First passage of non-Markov processes	319
7. Markov processes with large jumps	322
XIII. UNSTABLE SYSTEMS	
1. The bistable system	326
2. The escape time	333
3. Splitting probability	337
4. Diffusion in more dimensions	341
5. Critical fluctuations	344
6. Kramers' escape problem	347
7. Limit cycles and fluctuations	355
XIV. FLUCTUATIONS IN CONTINUOUS SYSTEMS	
1. Introduction	363
2. Diffusion noise	365
3. The method of compounding moments	367

4. Fluctuations in phase space density	371
5. Fluctuations and the Boltzmann equation	374
XV. THE STATISTICS OF JUMP EVENTS	
1. Basic formulae and a simple example	383
2. Jump events in nonlinear systems	386
3. Effect of incident photon statistics	388
4. Effect of incident photon statistics – continued	392
XVI. STOCHASTIC DIFFERENTIAL EQUATIONS	
1. Definitions	396
2. Heuristic treatment of multiplicative equations	399
3. The cumulant expansion introduced	405
4. The general cumulant expansion	407
5. Nonlinear stochastic differential equations	410
6. Long correlation times	416
XVII. STOCHASTIC BEHAVIOR OF QUANTUM SYSTEMS	
1. Quantum probability	422
2. The damped harmonic oscillator	428
3. The elimination of the bath	436
4. The elimination of the bath – continued	440
5. The Schrödinger–Langevin equation and the quantum master equation	444
6. A new approach to noise	449
7. Internal noise	451
SUBJECT INDEX	457