

Contents

	<i>How To Use This Book</i>	page xi
	<i>Acknowledgments</i>	xiv
Part I	Introduction	1
1	Concepts in Quantum Shannon Theory	3
	1.1 Overview of the Quantum Theory	7
	1.2 The Emergence of Quantum Shannon Theory	11
2	Classical Shannon Theory	26
	2.1 Data Compression	26
	2.2 Channel Capacity	35
	2.3 Summary	49
Part II	The Quantum Theory	51
3	The Noiseless Quantum Theory	53
	3.1 Overview	54
	3.2 Quantum Bits	55
	3.3 Reversible Evolution	61
	3.4 Measurement	68
	3.5 Composite Quantum Systems	74
	3.6 Summary and Extensions to Qudit States	89
	3.7 History and Further Reading	96
4	The Noisy Quantum Theory	97
	4.1 Noisy Quantum States	98
	4.2 Measurement in the Noisy Quantum Theory	110
	4.3 Composite Noisy Quantum Systems	112
	4.4 Noisy Evolution	120
	4.5 Summary	139
	4.6 History and Further Reading	140
5	The Purified Quantum Theory	141
	5.1 Purification	142
	5.2 Isometric Evolution	143

5.3	Coherent Quantum Instrument	154
5.4	Coherent Measurement	155
5.5	History and Further Reading	156
Part III	Unit Quantum Protocols	157
6	Three Unit Quantum Protocols	159
6.1	Non-local Unit Resources	160
6.2	Protocols	162
6.3	Optimality of the Three Unit Protocols	171
6.4	Extensions for Quantum Shannon Theory	173
6.5	Three Unit Qudit Protocols	174
6.6	History and Further Reading	180
7	Coherent Protocols	181
7.1	Definition of Coherent Communication	182
7.2	Implementations of a Coherent Bit Channel	184
7.3	Coherent Dense Coding	185
7.4	Coherent Teleportation	187
7.5	The Coherent Communication Identity	189
7.6	History and Further Reading	190
8	The Unit Resource Capacity Region	191
8.1	The Unit Resource Achievable Region	191
8.2	The Direct Coding Theorem	195
8.3	The Converse Theorem	196
8.4	History and Further Reading	200
Part IV	Tools of Quantum Shannon Theory	201
9	Distance Measures	203
9.1	Trace Distance	204
9.2	Fidelity	212
9.3	Relationships between Trace Distance and Fidelity	219
9.4	Gentle Measurement	223
9.5	Fidelity of a Noisy Quantum Channel	226
9.6	The Hilbert-Schmidt Distance Measure	230
9.7	History and Further Reading	231
10	Classical Information and Entropy	232
10.1	Entropy of a Random Variable	233
10.2	Conditional Entropy	237
10.3	Joint Entropy	239
10.4	Mutual Information	239
10.5	Relative Entropy	240

10.6	Conditional Mutual Information	241
10.7	Information Inequalities	243
10.8	Classical Information and Entropy of Quantum Systems	249
10.9	History and Further Reading	251
11	Quantum Information and Entropy	252
11.1	Quantum Entropy	253
11.2	Joint Quantum Entropy	258
11.3	Potential yet Unsatisfactory Definitions of Conditional Quantum Entropy	261
11.4	Conditional Quantum Entropy	263
11.5	Coherent Information	265
11.6	Quantum Mutual Information	267
11.7	Conditional Quantum Mutual Information	270
11.8	Quantum Relative Entropy	272
11.9	Quantum Information Inequalities	275
11.10	History and Further Reading	290
12	The Information of Quantum Channels	292
12.1	Mutual Information of a Classical Channel	293
12.2	Private Information of a Wiretap Channel	299
12.3	Holevo Information of a Quantum Channel	303
12.4	Mutual Information of a Quantum Channel	309
12.5	Coherent Information of a Quantum Channel	314
12.6	Private Information of a Quantum Channel	319
12.7	Summary	325
12.8	History and Further Reading	326
13	Classical Typicality	327
13.1	An Example of Typicality	328
13.2	Weak Typicality	329
13.3	Properties of the Typical Set	331
13.4	Application of Typical Sequences: Shannon Compression	333
13.5	Weak Joint Typicality	335
13.6	Weak Conditional Typicality	338
13.7	Strong Typicality	341
13.8	Strong Joint Typicality	350
13.9	Strong Conditional Typicality	352
13.10	Application: Shannon's Channel Capacity Theorem	358
13.11	Concluding Remarks	362
13.12	History and Further Reading	363
14	Quantum Typicality	364
14.1	The Typical Subspace	365
14.2	Conditional Quantum Typicality	375

14.3	The Method of Types for Quantum Systems	384
14.4	Concluding Remarks	387
14.5	History and Further Reading	387
15	The Packing Lemma	388
15.1	Introductory Example	389
15.2	The Setting of the Packing Lemma	389
15.3	Statement of the Packing Lemma	391
15.4	Proof of the Packing Lemma	393
15.5	Derandomization and Expurgation	398
15.6	History and Further Reading	400
16	The Covering Lemma	401
16.1	Introductory Example	402
16.2	Setting and Statement of the Covering Lemma	404
16.3	Proof of the Covering Lemma	406
16.4	History and Further Reading	413
Part V	Noiseless Quantum Shannon Theory	415
17	Schumacher Compression	417
17.1	The Information-Processing Task	418
17.2	The Quantum Data-Compression Theorem	420
17.3	Quantum Compression Example	424
17.4	Variations on the Schumacher Theme	425
17.5	Concluding Remarks	427
17.6	History and Further Reading	427
18	Entanglement Concentration	429
18.1	An Example of Entanglement Concentration	430
18.2	The Information-Processing Task	433
18.3	The Entanglement Concentration Theorem	433
18.4	Common Randomness Concentration	440
18.5	Schumacher Compression versus Entanglement Concentration	441
18.6	Concluding Remarks	445
18.7	History and Further Reading	445
Part VI	Noisy Quantum Shannon Theory	447
19	Classical Communication	451
19.1	Naive Approach: Product Measurements at the Decoder	453
19.2	The Information-Processing Task	456
19.3	The Classical Capacity Theorem	458
19.4	Examples of Channels	463

19.5	Superadditivity of the Holevo Information	471
19.6	Concluding Remarks	474
19.7	History and Further Reading	475
20	Entanglement-Assisted Classical Communication	477
20.1	The Information-Processing Task	479
20.2	A Preliminary Example	480
20.3	The Entanglement-Assisted Classical Capacity Theorem	484
20.4	The Direct Coding Theorem	484
20.5	The Converse Theorem	493
20.6	Examples of Channels	501
20.7	Concluding Remarks	506
20.8	History and Further Reading	507
21	Coherent Communication with Noisy Resources	508
21.1	Entanglement-Assisted Quantum Communication	509
21.2	Quantum Communication	514
21.3	Noisy Super-Dense Coding	515
21.4	State Transfer	518
21.5	Trade-off Coding	522
21.6	Concluding Remarks	530
21.7	History and Further Reading	531
22	Private Classical Communication	532
22.1	The Information-Processing Task	533
22.2	The Private Classical Capacity Theorem	536
22.3	The Direct Coding Theorem	536
22.4	The Converse Theorem	545
22.5	Discussion of Private Classical Capacity	546
22.6	History and Further Reading	549
23	Quantum Communication	550
23.1	The Information-Processing Task	551
23.2	The No-Cloning Theorem and Quantum Communication	553
23.3	The Quantum Capacity Theorem	554
23.4	The Direct Coding Theorem	555
23.5	Converse Theorem	562
23.6	An Interlude with Quantum Stabilizer Codes	564
23.7	Example Channels	571
23.8	Discussion of Quantum Capacity	574
23.9	Entanglement Distillation	579
23.10	History and Further Reading	582

24	Trading Resources for Communication	585
	24.1 The Information-Processing Task	586
	24.2 The Quantum Dynamic Capacity Theorem	588
	24.3 The Direct Coding Theorem	593
	24.4 The Converse Theorem	596
	24.5 Examples of Channels	606
	24.6 History and Further Reading	616
25	Summary and Outlook	618
	25.1 Unit Protocols	619
	25.2 Noiseless Quantum Shannon Theory	619
	25.3 Noisy Quantum Shannon Theory	620
	25.4 Protocols Not Covered in This Book	623
	25.5 Network Quantum Shannon Theory	624
	25.6 Future Directions	625
Appendix A	Miscellaneous Mathematics	626
Appendix B	Monotonicity of Quantum Relative Entropy	633
	<i>References</i>	639
	<i>Index</i>	653