Table of Contents

1 Markov Chains 1

1.1 Markov chains having two states 2
1.2 Transition function and initial distribution 5
1.3 Examples 6
1.4 Computations with transition functions 12
1.4.1 Hitting times 14
1.4.2 Transition matrix 16
1.5 Transient and recurrent states 17
1.6 Decomposition of the state space 21
1.6.1 Absorption probabilities 25
1.6.2 Martingales 27
1.7 Birth and death chains 29
1.8 Branching and queuing chains 33
1.8.1 Branching chain 34
1.8.2 Queuing chain 36
Appendix
1.9 Proof of results for the branching and queuing chains 36
1.9.1 Branching chain 38
1.9.2 Queuing chain 39

2 Stationary Distributions of a Markov Chain 47

2.1 Elementary properties of stationary distributions 47
2.2 Examples 49
2.2.1 Birth and death chain 50
2.2.2 Particles in a box 53
2.3 Average number of visits to a recurrent state 56
2.4 Null recurrent and positive recurrent states 60
2.5 Existence and uniqueness of stationary distributions 63
2.5.1 Reducible chains 67
2.6 Queuing chain 69
2.6.1 Proof 70

2.7	Convergence to the stationary distribution		72
	Appendix		
2.8	Proof of convergence		75
	2.8.1	Periodic case	77
	2.8.2	A result from number theory	79
3	**Markov Pure Jump Processes**		84
3.1	Construction of jump processes		84
3.2	Birth and death processes		89
	3.2.1	Two-state birth and death process	92
	3.2.2	Poisson process	94
	3.2.3	Pure birth process	98
	3.2.4	Infinite server queue	99
3.3	Properties of a Markov pure jump process		102
	3.3.1	Applications to birth and death processes	104
4	**Second Order Processes**		111
4.1	Mean and covariance functions		111
4.2	Gaussian processes		119
4.3	The Wiener process		122
5	**Continuity, Integration, and Differentiation of Second Order Processes**		128
5.1	Continuity assumptions		128
	5.1.1	Continuity of the mean and covariance functions	128
	5.1.2	Continuity of the sample functions	130
5.2	Integration		132
5.3	Differentiation		135
5.4	White noise		141
6	**Stochastic Differential Equations, Estimation Theory, and Spectral Distributions**		152
6.1	First order differential equations		154
6.2	Differential equations of order n		159
	6.2.1	The case $n = 2$	166
6.3	Estimation theory		170
	6.3.1	General principles of estimation	173
	6.3.2	Some examples of optimal prediction	174
6.4	Spectral distribution		177
Answers to Exercises			190
Glossary of Notation			199
Index			201