

CONTENTS

Introduction	xi
CHAPTER I. Groups and Counting Principles	1
1 Groups	1
2 G -spaces	2
3 Direct and semidirect products	5
4 Finite groups of rotations	11
5 The Platonic groups	13
6 The Sylow theorems	16
7 Counting and group structure	18
CHAPTER II. Fundamentals of Group Representations	21
1 Definition and unitarity	21
2 Irreducibility and complete reduction	23
3 The group algebra and the regular representations	25
4 Schur's lemma	27
5 Tensor products	29
6 Complex conjugate representations; Quaternionic representations	30
7 One-dimensional representations	34
CHAPTER III. Abstract Theory of Representations of Finite Groups	35
1 Orthogonality relations and the first fundamental relation	36
2 Characters, class functions, and conjugacy classes	39
3 One-dimensional representations	42
4 The dimension theorem	43
5 The theorem of Frobenius and Schur	47
Appendix to III.5—Representations on real and quaternionic vector spaces	50
6 Representations and group structure	55
7 Projections in the group algebra	56
8 Fourier analysis	57
9 Direct products	59
10 Restrictions	59
11 Subgroups of index 2	60
12 Examples	62

CHAPTER IV. Representations of Concrete Finite Groups. I: Abelian and Clifford Groups	65
1 The structure of finite abelian groups	65
2 Representations of abelian groups	67
3 The Clifford group	68
CHAPTER V. Representations of Concrete Finite Groups. II: Semidirect Products and Induced Representations	77
1 Frobenius theory of semidirect products	77
2 Examples of the semidirect product theory	81
3 Induced representations	83
4 The Frobenius character formula	85
5 The Frobenius reciprocity theorem	89
6 Mackey irreducibility criterion	91
7 Semidirect products, revisited	93
CHAPTER VI. Representations of Concrete Finite Groups. III: The Symmetric Groups	95
1 Permutations and classes	95
2 Young frames and Young tableaux	96
3 Projections in $\mathcal{A}(S_n)$: Classification of representations	101
4 Branching relations	108
5 The Frobenius character formula	109
6 Consequences of the character formula	117
CHAPTER VII. Compact Groups	121
1 C^∞ -manifolds: A review	121
2 Lie groups and Lie algebras	128
3 Haar measure on Lie groups	133
4 Matrix groups	135
5 The classical groups	137
6 Homotopy and covering groups	146
7 Spin groups	152
8 The structure of compact groups	155
9 Representations of compact groups: Abstract theory	155
10 The Peter-Weyl theorem	158
CHAPTER VIII. The Structure of Compact Semisimple Groups	165
1 Maximal tori	165
2 The Killing form	170
3 Representations of tori	173
4 Representations of $SU(2)$ and $\mathfrak{sl}(2, \mathbb{C})$	174
5 Roots and root spaces	177
6 Fundamental systems and their classification	183
7 Regular and singular elements	189

8 The Weyl group	192
9 The classical groups	196
CHAPTER IX. The Representations of Compact Semisimple Groups	205
1 Geometry of the Cartan-Stiefel diagram	206
2 The geometry of integral forms	210
3 The Weyl integration formula	213
4 Maximal weights	215
5 The classification theorem and the Weyl character formula	217
6 Consequences of the Weyl character formula	219
7 Representation theory: The algebraic approach	225
8 Representations of the classical groups	227
9 Determinant formulas for the classical characters	237
10 Real and quaternionic representations of the classical groups	242
11 Tensors, permutations, and the Frobenius character formula	246
Appendices	
A Multilinear algebra	253
B The analysis of self-adjoint Hilbert-Schmidt operators	257
Bibliography	261
Index	263