## Contents

	Preface	v
	CHAPTER 1 ONE-DIMENSIONAL FLOW	
1.1	Introduction	1
1.2	Eulerian Method	
	1.2.1 Flow along a Stream Tube	3
	1.2.2 Buoyancy Effect	6
1.3	The Pressure Equation	7
	1.3.1 Pitot Tubes	7
	1.3.2 Venturi Tube	9
	1.3.3 Manometer Readings	10
	1.3.4 Assessment of 'Incompressible' Assumption	11
1.4	Euler's Momentum Theorem for Steady Motion	12
	1.4.1 Force on a Stream Tube	12
	1.4.2 Sudden Enlargement of a Pipe	14
	Problems	15
	CHAPTER 2 GENERAL EQUATIONS OF MOTION	
2.1	Introduction	18
	2.1.1 Acceleration Vector	18
	2.1.2 Moving Axes	19
2.2	The Equations of Motion of an Inviscid Fluid	
	2.2.1 Rate of Change of Fluid Characteristics	21
	2.2.2 Continuity of Mass Flow	23
	2.2.3 Euler's Equation of Motion	24
2.3	Irrotational Motion	
	2.3.1 Bernoulli's Equation	25
	2.3.2 Persistence of Irrotational Motion	27
	2.3.3 Irrotational Motion—Pictorial Description	29
	2.3.4 Motion of a Fluid Element	29
	ix	

x	CONTENTS				
24	Boundary Conditions	32			
2.1	Uniqueness	32			
2.2	2.5.1 Kinetic Energy	32			
	2.5.2 Uniqueness Theorem	33			
26	Fuler's Momentum Theorem	34			
2.0	2.6.1 Force Exerted on Fixed Bodies	34			
	2.6.2 Theorem of Moment of Momentum	37			
	Problems	39			
	CHAPTER 3 TWO-DIMENSIONAL MOTION				
2 1	Introduction	42			
3.1	The Dimensional Functions	43			
3.2	2.2.1 Stream Function	43			
	3.2.1 Stream Function 3.2.2 Velocity Potential	44			
	3.2.2 Complex Potential	45			
	3.2.4 Indirect Approach	46			
	3.2.5 Inverse Function	40			
3.3	Basic Singularities	47			
0.0	3.3.1 Source	47 48			
	3.3.2 Doublet, Double Source or Dipole	49			
	3.3.3 Vortex	50			
	3.3.4 Mixed Flow	52			
3.4	Rankine Technique	53			
	3.4.1 Source + Sink	54			
	3.4.2 Source + Sink + Onnorm Stream	54			
	3.4.3 Stagnation Streamine				
3.5	Method of Images	56			
	2.5.2 Flow Past Circular Cylinder with Circulation	57			
	3.5.2 Flow Last Checkland Off	59			
2	Conformal Transformation	60			
3.0	2.6.1 Transformation of Flow Field	62			
	3.6.2 Transformation of Source	63			
	3.6.3 Transformation of Doublet	64			
2	7 The Aerofoil	65			
5.	3.7.1 The Joukowski Transformation	65			
	3.7.2 Thin Aerofoils	66 47			
	3.7.3 Joukowski's Hypothesis	07 60			
	3.7.4 Blasius's Theorem	09			

CONTENTS	
ing	

xi

70

	3.7.5 Lift Force	70
3.8	Potential Field Mapping	72
	3.8.1 Schwartz-Christoffel Theorem	73
	3.8.2 Semi-Infinite Strip	74
• •	3.8.3 Infinite Strip	75
3.9	Auxiliary Mapping Functions	76
	3.9.1 Zhukovski Function	76
2 10	Floatrical Applement	79
5.10	3 10.1 Boundary Conditions	83 84
	3.10.2 Representation of Circulation	85
	Problems	86
		00
	CHAPTER 4 IRROTATIONAL MOTION IN THREE	
	DIMENSIONS	
4.1	Introduction	91
4.2	Laplace's Equation	91
	4.2.1 Spherical Harmonics	91
	4.2.2 Axially Symmetric Field	92
	4.2.3 Stokes's Stream Function	93
	4.2.4 Motion of a Sphere	95
	4.2.6 Drag Force	95 97
4.3	Axial Distributions of Sources and Doublets	00
	4.3.1 Continuous Distributions	99
	4.3.2 Flow near Axis due to Sources	101
	4.3.3 Flow near Axis due to Doublets	102
4.4	Slender Bodies of Revolution	103
	4.4.1 Slender Body of Revolution in Uniform Stream	104
	4.4.2 Nose-up Pitching Moment	106
4.5	Motion Regarded as due to Sources and Doublets	108
	4.5.1 Green's Theorem and its Interpretation 4.5.2 Surfaces of Discontinuity	108
46	Alternative Depresentation	110
0	4.6.1 Vortex Sheets	111
	4.6.2 Vortex Density	111 114
	4.6.3 Induced Velocity	116
	4.6.4 Induced Drag	117
	Problems	119

# CHAPTER 5 DYNAMICS OF REAL FLUIDS

	•		123
5.1	Introd	uction	123
	5.1.1	Viscosity and Reynolds Number upon Flow Con-	
	5.1.2	Influence of Reynolds (value) apoint 2 and	125
	- 1 0	nguration Deal and Ideal Fluids	127
	5.1.3	Real and Ideal Fluid	131
	5.1.4	Secondary Flow	134
	5.1.5	Secondary How	135
5.2	The F	Equations of Wotion for Viscous Fluid	135
	5.2.1	The Stress Tensor III a viscous Trand	138
	5.2.2	Verticity and Circulation in a Viscous Fluid	149
	5.2.3	Vorticity and Chednation in a stores Equations	151
5.3	Some	Exact Solutions of the Navier-Stokes inquitions	
	5.3.1	Steady Flow through an Arbitrary Cymreol and	152
		Charles Countre Flow between Cylinders in Relative	
	5.3.2	Motion	156
	- 2 2	Stordy Flow between Parallel Planes	157
	5.3.3	Flow due to a Rotating Disc	160
	5.3.4	Some Solutions for Unsteady Flows	165
- 4	3.3.3 Maria	Slow Motion	167
5.4	very	Stokes's Flow	167
	5.4.1	Oseen Flow	171
	J.T.2		172
	Prob	lems	
	Сца	PTER 6 THE LAMINAR BOUNDARY LAYER IN	
	INC	OMPRESSIBLE FLOW	
	INC		175
6.1	Intro	oduction	175
	6.1.1	The Concept of the Boundary Eager	179
	6.1.2	Boundary-Layer Separation	181
6.2	2 The	Boundary Layer Equations	181
	6.2.1	Two-dimensional Flow along a Flane Wall	183
	6.2.2	2 Flow along a Curved Surface	184
	6.2.3	5 Some Important Boundary Layer Characteristics	187
	6.2.4	t Integral Equations of the Boundary 2-1,	190
	0.2.	y on whice of the Doundary Laver Equations	191
6.	3 Ana	lytic Solutions of the boundary Dayor Equations	192
	6.3.	Flow Parallel to a Semi-infinite Flat Flat	196
	6.3.	Z Flow near to the Stagnation Fond of a Symptot	

#### CONTENTS

xiii

	6.3.3	The Falkner–Skan Solutions	198
	6.3.4	Solution for a Linearly Decreasing External Velocity	199
	6.3.5	Solution for a Flow with Stagnation Point and Separation	203
6.4	Practi 6.4.1 6.4.2 6.4.3	cal Methods of Solving the Boundary-Layer Equations Pohlhausen's Method 'Thwaites's Method Stratford's Method	207 207 211 217
6.5	Some	Miscellaneous Boundary-Layer Problems	223
	6.5.1	Flow in Laminar Wakes and Jets	223
	6.5.2	Boundary Layers with Suction	226
	6.5.3	Flow in a Converging Channel	228
	Proble	ems	229

### CHAPTER 7 TURBULENT FLOW

7.1		Introduction		232
		7.1.1	The Mechanism of Transition to Turbulence	233
		7.1.2	The Essential Characteristics of Turbulence	235
		7.1.3	Reynolds Equations for Turbulent Motion	236
		7.1.4	Turbulent Flow between Parallel Planes	238
	7.2	Mixin	g-Length Theories of Turbulence	240
		7.2.1	Prandtl's Momentum-Transfer Theory	240
		7.2.2	The Mean Velocity in a Two-Dimensional	
			Turbulent Jet	241
		7.2.3	The Limitations of Mixing-Length Theories	244
	7.3	Turbu	ulent Boundary Layers	244
		7.3.1	The Two-Dimensional Turbulent Boundary Layer	244
		7.3.2	Flow in the Absence of Pressure Gradient	249
		7.3.3	General Calculations of Boundary-Layer Develop-	
			ment	251
		7.3.4	Flow near to Separation: Stratford's Analysis	256
	7.4	Corre	lation Theory of Homogeneous Turbulence	261
		7.4.1	Theoretical Deductions from the Navier-Stokes	
			Equations	262
		7.4.2	Isotropic Turbulence	265
		7.4.3	Eddy Sizes and Energy Dissipation	267

#### CONTENTS

v			
	7.4.4	The Momentum Equation in Isotropic Turbulence	269
	7.4.5	Some Deductions from the Kármán–Howarth Equation	271
7.5	Spectr 7.5.1 7.5.2 7.5.3 7.5.4 7.5.5 7.5.6	cal Theory of Homogeneous Turbulence The Energy Spectrum Tensor Spectra in Isotropic Turbulence The Relationship between $E(K)$ and $f(r)$ Rate of Change of the Energy Spectrum The Probability Distribution of $u(\mathbf{x})$ Calculation of the Pressure Covariance in Isotropic Turbulence	<ul> <li>273</li> <li>273</li> <li>274</li> <li>275</li> <li>278</li> <li>279</li> <li>280</li> </ul>
	Probl	ems	282
			285
	Kefer	ences	287
	Index	ζ	

• •

xiv