

Contents – Volume I

<i>Preface</i>	ix
<i>Introduction</i>	1
1. Introduction to Classical Computation	9
1.1 The Turing machine	9
1.1.1 Addition on a Turing machine	12
1.1.2 The Church–Turing thesis	13
1.1.3 The universal Turing machine	14
1.1.4 The probabilistic Turing machine	14
1.1.5 * The halting problem	15
1.2 The circuit model of computation	15
1.2.1 Binary arithmetics	17
1.2.2 Elementary logic gates	17
1.2.3 Universal classical computation	22
1.3 Computational complexity	24
1.3.1 Complexity classes	27
1.3.2 * The Chernoff bound	30
1.4 * Computing dynamical systems	30
1.4.1 * Deterministic chaos	31
1.4.2 * Algorithmic complexity	33
1.5 Energy and information	35
1.5.1 Maxwell’s demon	35
1.5.2 Landauer’s principle	37
1.5.3 Extracting work from information	40
1.6 Reversible computation	41

1.6.1	Toffoli and Fredkin gates	43
1.6.2	* The billiard-ball computer	45
1.7	A guide to the bibliography	47
2.	Introduction to Quantum Mechanics	49
2.1	The Stern–Gerlach experiment	50
2.2	Young’s double-slit experiment	53
2.3	Linear vector spaces	57
2.4	The postulates of quantum mechanics	76
2.5	The EPR paradox and Bell’s inequalities	88
2.6	A guide to the bibliography	97
3.	Quantum Computation	99
3.1	The qubit	100
3.1.1	The Bloch sphere	102
3.1.2	Measuring the state of a qubit	103
3.2	The circuit model of quantum computation	105
3.3	Single-qubit gates	108
3.3.1	Rotations of the Bloch sphere	110
3.4	Controlled gates and entanglement generation	112
3.4.1	The Bell basis	118
3.5	Universal quantum gates	118
3.5.1	* Preparation of the initial state	127
3.6	Unitary errors	130
3.7	Function evaluation	132
3.8	The quantum adder	137
3.9	Deutsch’s algorithm	140
3.9.1	The Deutsch–Jozsa problem	141
3.9.2	* An extension of Deutsch’s algorithm	143
3.10	Quantum search	144
3.10.1	Searching one item out of four	145
3.10.2	Searching one item out of N	147
3.10.3	Geometric visualization	149
3.11	The quantum Fourier transform	152
3.12	Quantum phase estimation	155
3.13	* Finding eigenvalues and eigenvectors	158
3.14	Period finding and Shor’s algorithm	161
3.15	Quantum computation of dynamical systems	164

3.15.1	Quantum simulation of the Schrödinger equation	164
3.15.2 *	The quantum baker's map	168
3.15.3 *	The quantum sawtooth map	170
3.15.4 *	Quantum computation of dynamical localization	174
3.16	First experimental implementations	178
3.16.1	Elementary gates with spin qubits	179
3.16.2	Overview of the first implementations	181
3.17	A guide to the bibliography	185
4.	Quantum Communication	189
4.1	Classical cryptography	189
4.1.1	The Vernam cypher	190
4.1.2	The public-key cryptosystem	191
4.1.3	The RSA protocol	192
4.2	The no-cloning theorem	194
4.2.1	Faster-than-light transmission of information?	197
4.3	Quantum cryptography	198
4.3.1	The BB84 protocol	199
4.3.2	The E91 protocol	202
4.4	Dense coding	204
4.5	Quantum teleportation	208
4.6	An overview of the experimental implementations	213
4.7	A guide to the bibliography	214
Appendix A	Solutions to the exercises	215
<i>Bibliography</i>		241
<i>Index</i>		253

Contents – Volume II

<i>Preface</i>	vii
5. Quantum Information Theory	257
5.1 The density matrix	258
5.1.1 The density matrix for a qubit	264
5.1.2 Composite systems	267
5.1.3 * The quantum copying machine	271
5.2 The Schmidt decomposition	273
5.3 Purification	276
5.4 The Kraus representation	278
5.5 Measurement of the density matrix for a qubit	284
5.6 Generalized measurements	286
5.6.1 * Weak measurements	288
5.6.2 POVM measurements	290
5.7 The Shannon entropy	293
5.8 Classical data compression	294
5.8.1 Shannon's noiseless coding theorem	294
5.8.2 Examples of data compression	296
5.9 The von Neumann entropy	297
5.9.1 Example 1: source of orthogonal pure states	299
5.9.2 Example 2: source of non-orthogonal pure states	300
5.10 Quantum data compression	303
5.10.1 Schumacher's quantum noiseless coding theorem	303
5.10.2 Compression of an n -qubit message	304
5.10.3 Example 1: two-qubit messages	306
5.10.4 Example 2: three-qubit messages	308

5.11 Accessible information	311
5.11.1 The Holevo bound	313
5.11.2 Example 1: two non-orthogonal pure states	313
5.11.3 * Example 2: three non-orthogonal pure states	317
5.12 Entanglement concentration and von Neumann entropy	319
5.13 The Peres separability criterion	323
5.14 * Entropies in physics	325
5.14.1 * Thermodynamic entropy	325
5.14.2 * Statistical entropy	328
5.14.3 * Dynamical Kolmogorov–Sinai entropy	330
5.15 A guide to the bibliography	333
 6. Decoherence	335
6.1 Decoherence models for a single qubit	336
6.1.1 The quantum black box	337
6.1.2 Measuring a quantum operation acting on a qubit	339
6.1.3 Quantum circuits simulating noise channels	340
6.1.4 The bit-flip channel	343
6.1.5 The phase-flip channel	344
6.1.6 The bit-phase-flip channel	345
6.1.7 The depolarizing channel	346
6.1.8 Amplitude damping	347
6.1.9 Phase damping	349
6.1.10 De-entanglement	351
6.2 The master equation	354
6.2.1 * Derivation of the master equation	355
6.2.2 * The master equation and quantum operations	359
6.2.3 The master equation for a single qubit	362
6.3 Quantum to classical transition	365
6.3.1 Schrödinger’s cat	365
6.3.2 Decoherence and destruction of cat states	367
6.4 * Decoherence and quantum measurements	375
6.5 * Quantum chaos	378
6.5.1 * Dynamical chaos in classical mechanics	379
6.5.2 * Quantum chaos and the correspondence principle	382
6.5.3 * Time scales of quantum chaos	385
6.5.4 * Quantum chaos and Anderson localization	392
6.5.5 * The hydrogen atom in a microwave field	395
6.5.6 * Quantum chaos and universal spectral fluctuations	400

6.5.7 * The chaos border for the quantum computer hardware	412
6.5.8 * The quantum Loschmidt echo	416
6.5.9 * Dynamical stability of quantum motion	423
6.5.10 * Dynamical chaos and dephasing: the double-slit experiment	425
6.5.11 * Entanglement and chaos	430
6.6 Decoherence and quantum computation	434
6.6.1 * Decoherence and quantum trajectories	439
6.7 * Quantum computation and quantum chaos	449
6.7.1 * Quantum versus classical errors	451
6.7.2 * Static imperfections versus noisy gates	452
6.8 A guide to the bibliography	457
 7. Quantum Error Correction	459
7.1 The three-qubit bit-flip code	461
7.2 The three-qubit phase-flip code	465
7.3 The nine-qubit Shor code	466
7.4 General properties of quantum error correction	471
7.4.1 The quantum Hamming bound	473
7.5 * The five-qubit code	474
7.6 * Classical linear codes	476
7.6.1 * The Hamming codes	478
7.7 * CSS codes	481
7.8 Decoherence-free subspaces	484
7.8.1 * Conditions for decoherence-free dynamics	486
7.8.2 * The spin-boson model	488
7.9 * The Zeno effect	490
7.10 Fault-tolerant quantum computation	494
7.10.1 Avoidance of error propagation	495
7.10.2 Fault-tolerant quantum gates	497
7.10.3 The noise threshold for quantum computation	497
7.11 * Quantum cryptography over noisy channels	500
7.12 * Quantum channels with memory	506
7.13 A guide to the bibliography	510
 8. First Experimental Implementations	513
8.1 NMR quantum computation	514
8.1.1 The system Hamiltonian	515

8.1.2 The physical apparatus	518
8.1.3 Quantum ensemble computation	519
8.1.4 Refocusing	522
8.1.5 Demonstration of quantum algorithms	523
8.2 Cavity quantum electrodynamics	528
8.2.1 Rabi oscillations	535
8.2.2 Entanglement generation	538
8.2.3 The quantum phase gate	542
8.3 The ion-trap quantum computer	544
8.3.1 The Paul trap	544
8.3.2 Laser pulses	547
8.3.3 Realization of the Cirac–Zoller CNOT gate	554
8.3.4 Entanglement generation	556
8.4 Solid state qubits	561
8.4.1 Spins in semiconductors	561
8.4.2 Quantum dots	562
8.4.3 Superconducting qubit circuits	566
8.5 Quantum communication with photons	573
8.5.1 Linear optics	573
8.5.2 Experimental quantum teleportation	578
8.5.3 Experimental quantum-key distribution	587
8.6 Problems and prospects	591
8.7 A guide to the bibliography	592
Appendix B Solutions to the exercises	595
<i>Bibliography</i>	657
<i>Index</i>	675